# Probabilistic Model Checking


Marta Kwiatkowska
Dave Parker

Oxford University Computing Laboratory


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# Course overview

- 5 lectures: Mon-Fri, 11am-12.30pm
  - Introduction
  - 1 – Discrete time Markov chains
  - 2 – Markov decision processes
  - 3 – Continuous-time Markov chains
  - 4 – Probabilistic model checking in practice
  - 5 – Probabilistic timed automata
- Course materials available here:
  - http://www.prismmodelchecker.org/lectures/esslli10/
  - lecture slides, reference list

# Part 4

## Probabilistic model checking in practice

# Overview (Part 4)

- Tool support for probabilistic model checking
  - motivation, existing tools
- The PRISM model checker
  - functionality, features
  - modelling language & property specification
  - PRISM demonstration
- Probabilistic counterexamples
  - (smallest) counterexamples for PCTL + DTMCs
- Probabilistic bisimulation
  - bisimulation equivalences for DTMCs, CTMCs + minimisation

# Motivation

- Complexity of PCTL model checking
  - generally polynomial in model size (number of states)
- State space explosion problem
  - models for realistic case studies are typically huge
- Clearly tool support is required
- Benefits:
  - fully automated process
  - high-level languages/formalisms for building models
  - visualisation of quantitative results

# Tools – Probabilistic model checkers

- PRISM (Probabilistic Symbolic Model Checker)
  - DTMCs, MDPs, CTMCs + rewards, [Birmingham/Oxford]
- MRMC (Markov Reward Model Checker)
  - DTMCs, CTMCs + reward extensions, [Twente/Aachen]
- LiQuor: LTL model checking for MDPs, Probmela language (probabilistic version of SPIN's Promela), [Dresden]

- Simulation-based probabilistic model checking:
  - APMC, Ymer (both based on PRISM language), VESTA
- Many other related tools/prototypes
  - RAPTURE, CADP, Möbius, APNN-Toolbox, SMART, GreatSPN, GRIP, CASPA, Premo, PASS, …

# The PRISM tool

- PRISM: Probabilistic symbolic model checker
  - developed at Birmingham/Oxford University, since 1999
  - free, open source (GPL)
  - versions for Linux, Unix, Mac OS X, Windows, 64-bit OSs

- Modelling of:
  - DTMCs, CTMCs, MDPs + costs/rewards

- Model checking of:
  - PCTL, CSL, LTL, PCTL* + extensions + costs/rewards

# PRISM functionality

- High-level modelling language
- Wide range of model analysis methods
  - efficient symbolic implementation techniques
  - also: approximate verification using simulation + sampling
- Graphical user interface
  - model/property editor
  - discrete-event simulator - model traces for debugging, etc.
  - easy automation of verification experiments
  - graphical visualisation of results
- Command-line version
  - same underlying verification engines
  - useful for scripting, batch jobs

# Modelling languages/formalisms

- Many high-level modelling languages, formalisms available
- For example:
  - probabilistic/stochastic process algebras
  - stochastic Petri nets
  - stochastic activity networks
- Custom languages for tools, e.g.:
  - PRISM modelling language
  - Probmela (probabilistic variant of Promela, the input language for the model checker SPIN) - used in LiQuor

# PRISM modelling language

- Simple, textual, state-based language
  - modelling of DTMCs, CTMCs and MDPs
  - based on Reactive Modules [AH99]

- Basic components…
- Modules:
  - components of system being modelled
  - composed in parallel
- Variables
  - finite (integer ranges or Booleans)
  - local or global
  - all variables public: anyone can read, only owner can modify

# PRISM modelling language

- Guarded commands
  - describe behaviour of each module
  - i.e. the changes in state that can occur
  - labelled with probabilities (or, for CTMCs, rates)
  - (optional) action labels

[send] (s=2) -> $p_{loss}$ : (s'=3)&(lost'=lost+1) + (1-$p_{loss}$) : (s'=4);

action guard probability update probability update

# PRISM modelling language

- Parallel composition
  - model multiple components that can execute independently
  - for DTMC models, mostly assume components operate synchronously, i.e. move in lock-step
- Synchronisation
  - simultaneous transitions in more than one module
  - guarded commands with matching action-labels
  - probability of combined transition is product of individual probabilities for each component
- More complex parallel compositions can be defined
  - using process-algebraic operators
  - other types of parallel composition, action hiding/renaming

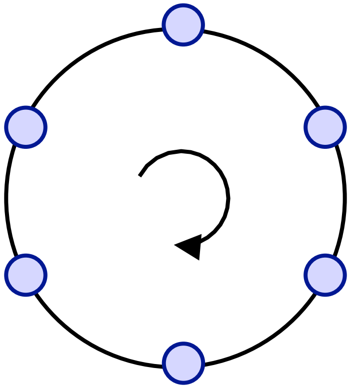
# Simple example

```
module M1
    x : [0..3] init 0;
    [a] x=0 -> (x'=1);
    [b] x=1 -> 0.5:(x'=2) + 0.5:(x'=3);
endmodule
```

```
module M2
    y : [0..3] init 0;
    [a] y=0 -> (y'=1);
    [b] y=1 -> 0.4:(y'=2) + 0.6:(y'=3);
endmodule
```

# Example: Leader election

- Randomised leader election protocol
  - due to Itai & Rodeh (1990)
- Set-up: N nodes, connected in a ring
  - communication is synchronous (lock-step)
- Aim: elect a leader
  - i.e. one uniquely designated node
  - by passing messages around the ring



- Protocol operates in rounds. In each round:
  - each node choose a (uniformly) random id $\in \{0,\ldots,k-1\}$
  - (k is a parameter of the protocol)
  - all nodes pass their id around the ring
  - if there is maximum unique id, node with this id is the leader
  - if not, try again with a new round

# PRISM code

# PRISM property specifications

- Based on (probabilistic extensions of) temporal logic
  - incorporates PCTL, CSL, LTL, PCTL*
  - also includes: quantitative extensions, costs/rewards
- Leader election properties
  - $P_{\geq 1}$ [ F elected ]
    - with probability 1, a leader is eventually elected
  - $P_{>0.8}$ [ $F^{\leq k}$ elected ]
    - with probability greater than 0.8, a leader is elected within k steps
- Usually focus on quantitative properties:
  - $P_{=?}$ [ $F^{\leq k}$ elected ]
    - what is the probability that a leader is elected within k steps?

# PRISM property specifications

- Best/worst-case scenarios
  - combining "quantitative" and "exhaustive" aspects

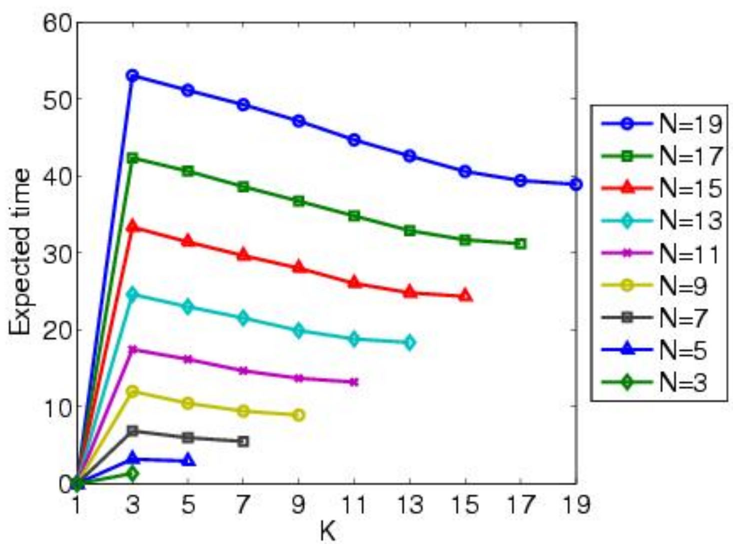
- e.g. computing values for a range of states…

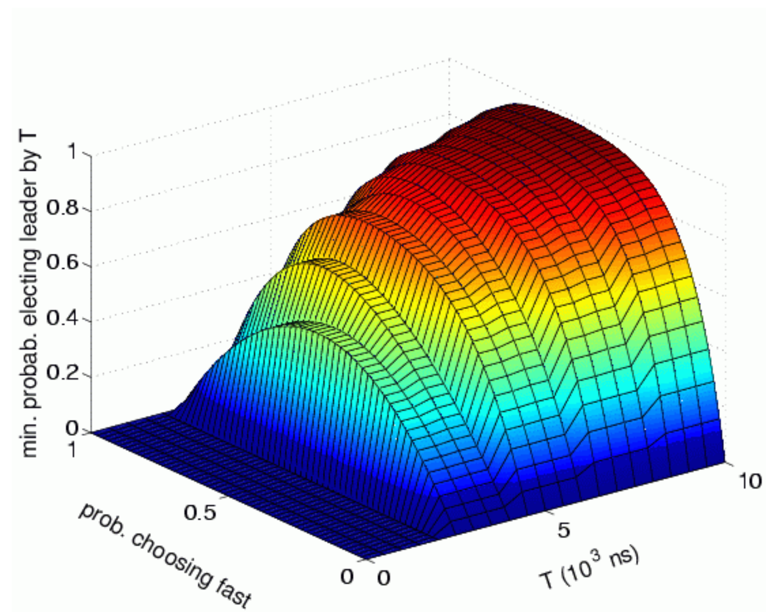
- $P_{=?}$ [ $F^{\leq t}$ elected {tokens≤k}{min} ] –
  - "minimum probability of the leader election algorithm completing within t steps from any state where there are at most k tokens"

- $R_{=?}$ [ F end {"init"}{max} ] –
  - "maximum expected run-time over all possible initial configurations"

# PRISM property specifications

- Experiments:
  - ranges of model/property parameters
  - e.g. $P_{=?}$ [ $F^{\leq T}$ error ] for N=1..5, T=1..100
    where N is some model parameter and T a time bound
  - identify patterns, trends, anomalies in quantitative results



# PRISM…

# More info on PRISM

- PRISM website: http://www.prismmodelchecker.org/
  - tool download: binaries, source code (GPL)
  - example repository (50+ case studies)
  - on-line PRISM manual
  - support: help forum, bug tracking, feature requests
  - related publications, talks, tutorials, links
- Tutorial: http://www.prismmodelchecker.org/tutorial/

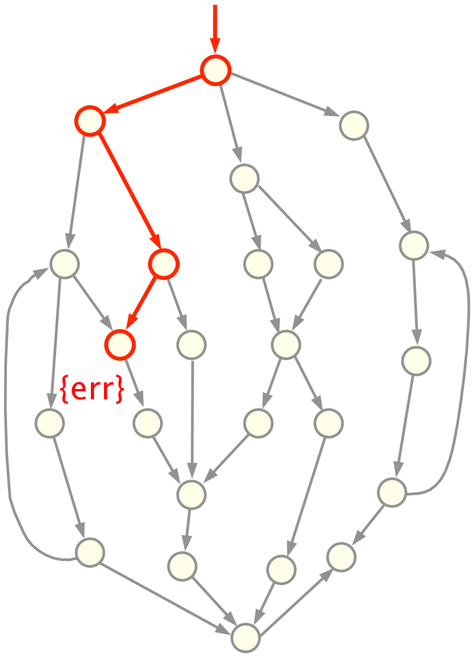
# Overview (Part 4)

- Tool support for probabilistic model checking
  - motivation, existing tools
- The PRISM model checker
  - functionality, features
  - modelling language & property specification
  - PRISM demonstration
- **Probabilistic counterexamples**
  - (smallest) counterexamples for PCTL + DTMCs
- Probabilistic bisimulation
  - bisimulation equivalences for DTMCs, CTMCs + minimisation

# Non probabilistic counterexamples

- Counterexamples (for non-probabilistic model checking)
  - generated when model checking a (universal) property fails
  - trace through model illustrating why property does not hold
  - major advantage of the model checking approach
  - bug finding vs. verification

- Example:
  - CTL property AG ¬err
  - (or equivalently, ¬EF err)
  - ("an error state is never reached")
  - counterexample is a finite trace to a state satisfying err
  - alternatively, this is a witness to the satisfaction of formula EF err



# Counterexamples for DTMCs?

- PCTL example: $P_{<0.01}$ [ F err ]
  - "the probability of reaching an error state is less than 0.01"
  - what is a counterexample for $s \nvDash P_{<0.01}$ [ F err ] ?
  - not necessarily illustrated by a single trace to an err state
  - in fact, "counterexample" is a set of paths satisfying F err whose combined measure is greater than or equal to 0.01

- Alternative approach seen so far:
  - probabilistic model checker provides actual probabilities
  - e.g. queries of the form $P_{=?}$ [ F err ]
  - anomalous behaviour identified by examining trends
  - e.g. $P_{=?}$ [ $F^{\leq T}$ err ] for T=0,…,100

- This lecture: DTMC counterexamples in style of [HK07]
  - also some work done on CTMC/MDP counterexamples

# DTMC notation

- DTMC: $D = (S, s_{init}, \mathbf{P}, L)$
- $Path(s)$ = set of all infinite paths starting in state s
- $Pr_s : \Sigma_{Path(s)} \rightarrow [0,1]$ = probability measure over infinite paths
  - where $\Sigma_{Path(s)}$ is the σ-algebra on $Path(s)$
  - defined in terms of probabilities for finite paths
- $\mathbf{P}_s(\omega)$ = probability for finite path $\omega = ss_1 \ldots s_n$
  - $\mathbf{P}_s(s) = 1$
  - $\mathbf{P}_s(ss_1 \ldots s_n) = \mathbf{P}(s,s_1) \cdot \mathbf{P}(s_1,s_2) \cdot \ldots \cdot \mathbf{P}(s_{n-1},s_n)$
  - extend notation to sets: $\mathbf{P}_s(C)$ for set of finite paths C
  - $\mathbf{P}_s$ extends uniquely to $Pr_s$
- $Path(s, \psi) = \{ \omega \in Path(s) \mid \omega \vDash \psi \}$
  - $Prob(s, \psi) = Pr_s(Path(s, \psi))$
- $Path_{fin}(s, \psi)$ = set of finite paths from s satisfying ψ

# Counterexamples for DTMCs

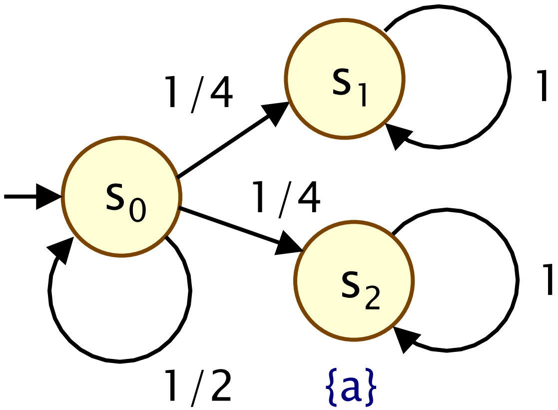
- Consider PCTL properties of the form:
  - $P_{\leq p}$ [ $\Phi_1$ $U^{\leq k}$ $\Phi_2$ ], where $k \in \mathbb{N} \cup \{\infty\}$
  - i.e. bounded or unbounded until formulae with closed upper probability bounds

- Refutation:
  - $s \not\models P_{\leq p}$ [ $\Phi_1$ $U^{\leq k}$ $\Phi_2$ ]
  - $\Leftrightarrow$ Prob(s, [ $\Phi_1$ $U^{\leq k}$ $\Phi_2$ ]) $>$ p
  - $\Leftrightarrow$ $Pr_s$(Path(s, $\Phi_1$ $U^{\leq k}$ $\Phi_2$)) $>$ p
  - i.e. total probability mass of $\Phi_1$ $U^{\leq k}$ $\Phi_2$ paths exceeds p

- Since the property is an until formula
  - this is evidenced by a set of finite paths

# Counterexamples for DTMCs

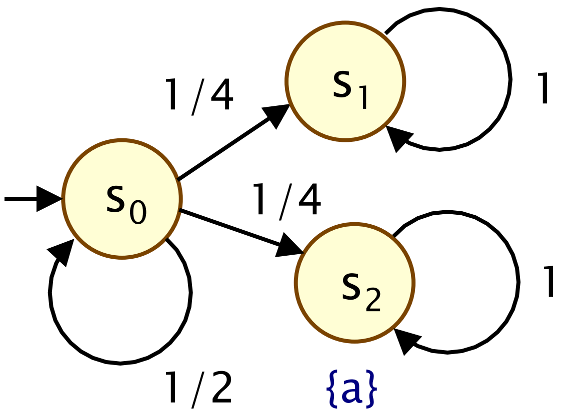
- A counterexample for $P_{\leq p}$ [ $\Phi_1$ $U^{\leq k}$ $\Phi_2$ ] in state s is:
  - a set C of finite paths such that $C \subseteq Path_{fin}(s, \psi)$ and $P_s(C) > p$




- Example
  - Consider the PCTL formula:
  - $P_{\leq 0.3}$ [ F a ]
  - This is not satisfied in $s_0$
  - $Prob(s_0, F\ a) = 1/4+1/8+1/16+\ldots = 1/2$
  - A counterexample: $C = \{ s_0s_2, s_0s_0s_2 \}$
  - $P_{s0}(C) = 1/4 + (1/2)(1/4) = 3/8 = 0.375$

# Finiteness of counterexamples

- There is always a finite counterexample for:
  - $s \not\models P_{\leq p}\,[\,\Phi_1\ U^{\leq k}\ \Phi_2\,]$
- On the other hand, consider this DTMC:
  - and the PCTL formula:
  - $P_{<1/2}\,[\,F\ a\,]$




  - $Prob(s_0, F\ a) = 1/4+1/8+1/16+\ldots$
    $= 1/2$
  - $s_0 \not\models P_{<1/2}\,[\,F\ a\,]$
  - counterexample would require infinite set of paths
  - $\{\,(s_0)^i s_2\,\}_{i\in\mathbb{N}}$

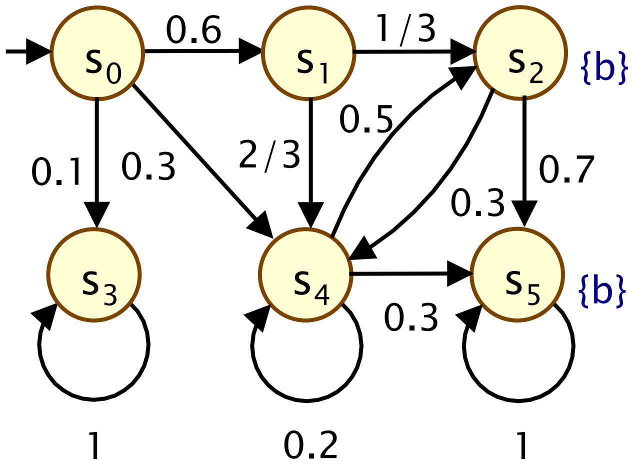
# Counterexamples for DTMCs

- Aim: counterexamples should be succinct, comprehensible

- Set of all counterexamples:
  - $CX_p(s,\psi)$ = set of all counterexamples for $P_{\leq p}$ [ψ] in state s
- Minimal counterexample
  - counterexample C with $|C| \leq |C'|$ for all $C' \in CX_p(s,\psi)$
- "Smallest" counterexample
  - minimal counterexample C with $\mathbf{P}(C) \geq \mathbf{P}(C')$ for all minimal $C' \in CX_p(s,\psi)$
- Strongest (most probable) evidence
  - finite path ω in $Path_{fin}(s, \psi)$ such that $\mathbf{P}(\omega) \geq \mathbf{P}(\omega')$ for all $\omega' \in Path_{fin}(s, \psi)$
  - i.e. contributes most to violation of PCTL formula

# Example

- PCTL formula: $P_{\leq 1/2}$ [ F b ]
  - $s_0 \not\models P_{\leq 1/2}$ [ F b ]
  - since Prob($s_0$, F b) = 0.9




- Counterexamples:
  - $C_1$ = { $s_0s_1s_2$, $s_0s_1s_4s_2$, $s_0s_1s_4s_5$, $s_0s_4s_2$ }
    - $P_{s0}(C_1)$ = 0.2+0.2+0.12+0.15 = 0.67 (not minimal)
  - $C_2$ = { $s_0s_1s_2$, $s_0s_1s_4s_2$, $s_0s_1s_4s_5$ }
    - $P_{s0}(C_2)$ = 0.2+0.2+0.12 = 0.52 (not "smallest")
  - $C_3$ = { $s_0s_1s_2$, $s_0s_1s_4s_2$, $s_0s_4s_2$ }
    - $P_{s0}(C_3)$ = 0.2+0.2+0.15 = 0.55

# Weighted digraphs

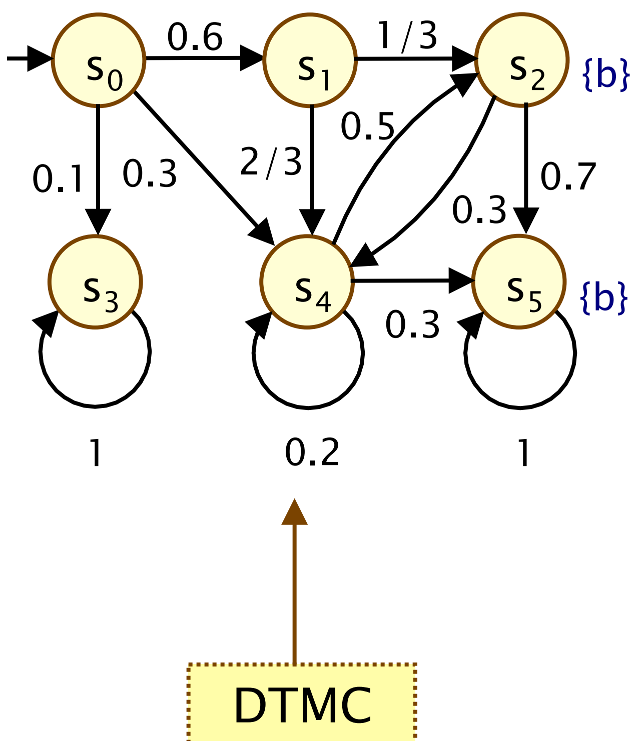
- A weighted directed graph is a tuple $G = (V, E, w)$ where:
  - $V$ is a set of vertices
  - $E \subseteq V \times V$ is a set of edges
  - $w : E \to \mathbb{R}_{\geq 0}$ is a weight function
- Finite path $\omega$ in $G$
  - is a sequence of vertices $v_0 v_1 v_2 \ldots v_n$ such that $(v_i, v_{i+1}) \in E \ \forall i \geq 0$
  - the distance of $\omega = v_0 v_1 v_2 \ldots v_n$ is: $\Sigma_{i=0\ldots n-1}\, w(v_i, v_{i+1})$
- Shortest path problem
  - given a weighted digraph, find a path between two vertices $v_1$ and $v_2$ with the smallest distance
  - i.e. a path $\omega$ s.t. $d(\omega) \leq d(\omega')$ for all other such paths $\omega'$

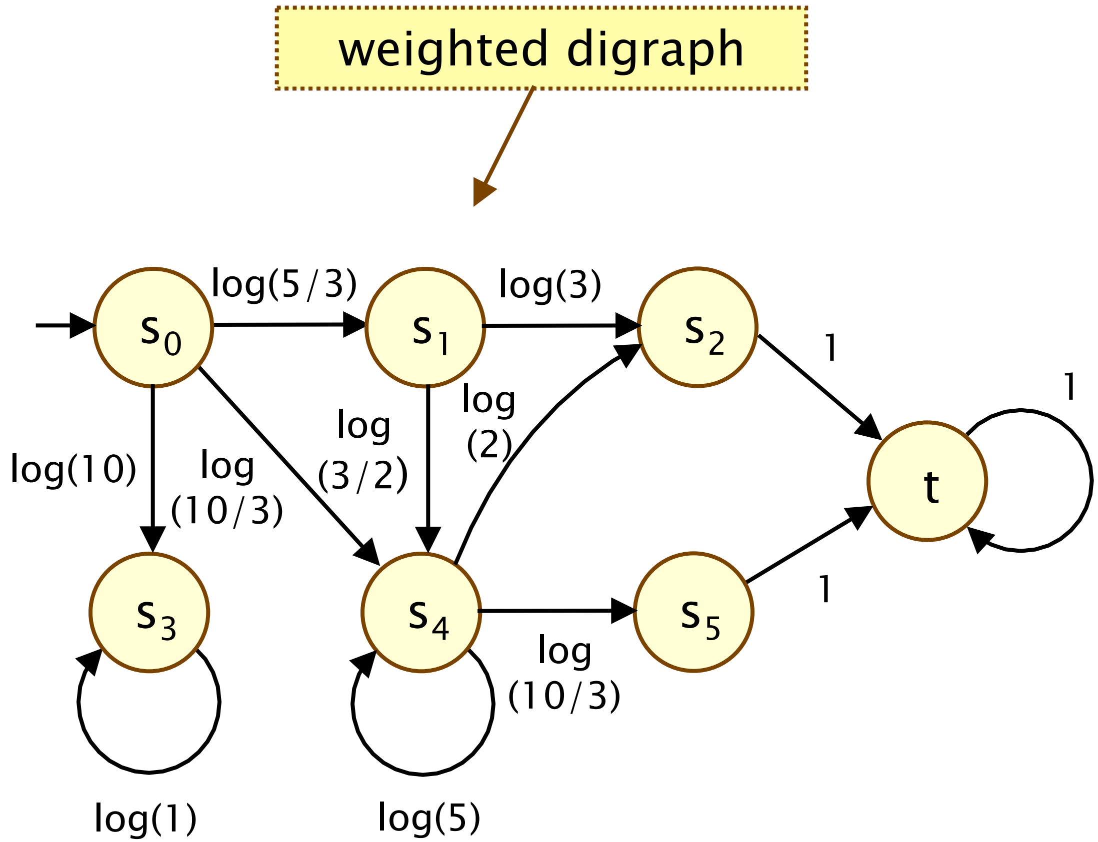
# Finding strongest evidences

- Reduction to graph problem…
- Step 1: Adapt the DTMC
  - make states satisfying $\neg\Phi_1 \wedge \neg\Phi_2$ absorbing
    - (i.e. replace all outgoing transitions with a single self-loop)
  - add an extra state t and replace all transitions from any $\Phi_2$ state with a single transition to t (with probability 1)
- Step 2: Convert new DTMC into a weighted digraph
  - for the (adapted) DTMC $D = (S, s_{init}, \mathbf{P}, L)$:
  - corresponding graph is $G_D = (V, E, w)$ where:
  - $V = S$ and $E = \{ (s,s') \in S \times S \mid \mathbf{P}(s,s') > 0 \}$
  - $w(s,s') = \log(1/\mathbf{P}(s,s'))$
- Key idea: for any two paths ω and ω' in D (and in $G_D$)
  - $\mathbf{P}_s(\omega') \geq \mathbf{P}_s\{\omega\}$ if and only if $d(\omega') \leq d(\omega)$

# Example…

- PCTL formula: $P_{\leq 1/2}$ [ F b ]



# Finding strongest evidences

- To find strongest evidence in DTMC D
  - analyse corresponding digraph
- For unbounded until formula $P_{\leq p}$ [ $\Phi_1$ U $\Phi_2$ ]
  - solve shortest path problem in digraph (target t)
  - polynomial time algorithms exist
    - e.g. Dijsktra's algorithm can be implemented in $O(|E|+|V|\cdot\log|V|)$
- For bounded until formula $P_{\leq p}$ [ $\Phi_1$ $U^{\leq k}$ $\Phi_2$ ]
  - solve special case of the constrained shortest path problem
  - also solvable in polynomial time
- Generation of smallest counterexamples
  - based on computation of k shortest paths
  - k can be computed on the fly

# Other cases

- Lower bounds on probabilities
  - i.e. $s \not\models P_{\geq p}\ [\ \Phi_1\ U^{\leq k}\ \Phi_2\ ]$
  - negate until formula to reverse probability bound
  - solvable with BSCC computation + probabilistic reachability
  - for details, see [HK07]

- Continuous-time Markov chains
  - these techniques can be extended to CTMCs and CSL [HK07b]
  - naïve approach: apply DTMC techniques to uniformised DTMC
  - modifications required to get smaller counterexamples
  - another possibility: directed search based techniques [AHL05]

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# Bisimulation

- Identifies models with the same branching structure
  - i.e. the same stepwise behaviour
  - each model can simulate the actions of the other
  - guarantees that models satisfy many of the same properties

- Uses of bisimulation:
  - show equivalence between a model and its specification
  - state space reduction: bisimulation minimisation

- Formally, bisimulation is an equivalence relation over states
  - bisimilar states must have identical labelling
    and identical stepwise behaviour

# Bisimulation on DTMCs

- Consider a DTMC D = $(S, s_{init}, \mathbf{P}, L)$

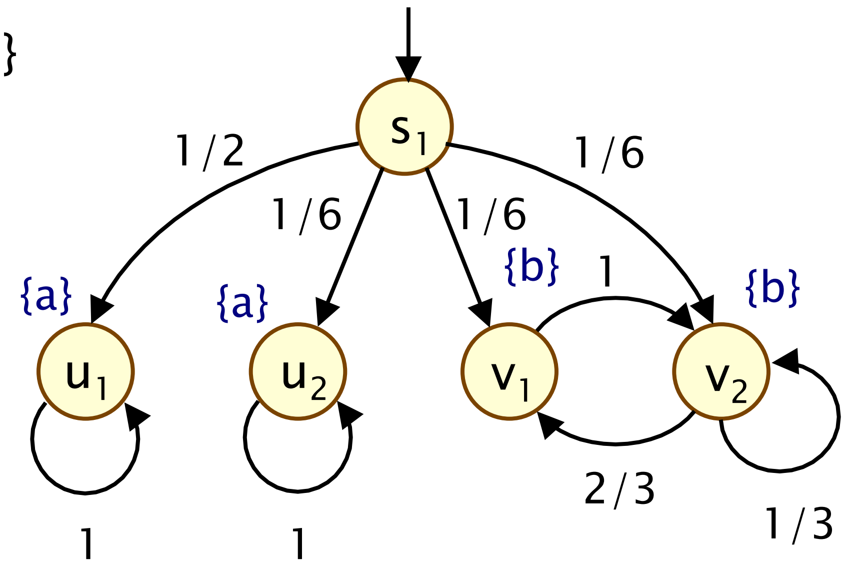
- Some notation:
  - $\mathbf{P}(s,T) = \Sigma_{s' \in T} \mathbf{P}(s,s')$ for $T \subseteq S$

- An equivalence relation R on S is a probabilistic bisimulation on D if and only if for all $s_1$ R $s_2$:
  - $L(s_1) = L(s_2)$
  - $\mathbf{P}(s_1, T) = \mathbf{P}(s_2, T)$ for all $T \in S/R$ (i.e. for all equivalence classes of R)

- States $s_1$ and $s_2$ are bisimulation-equivalent (or bisimilar)
  - if there exists a probabilistic bisimulation R on D with $s_1$ R $s_2$
  - denoted $s_1 \sim s_2$

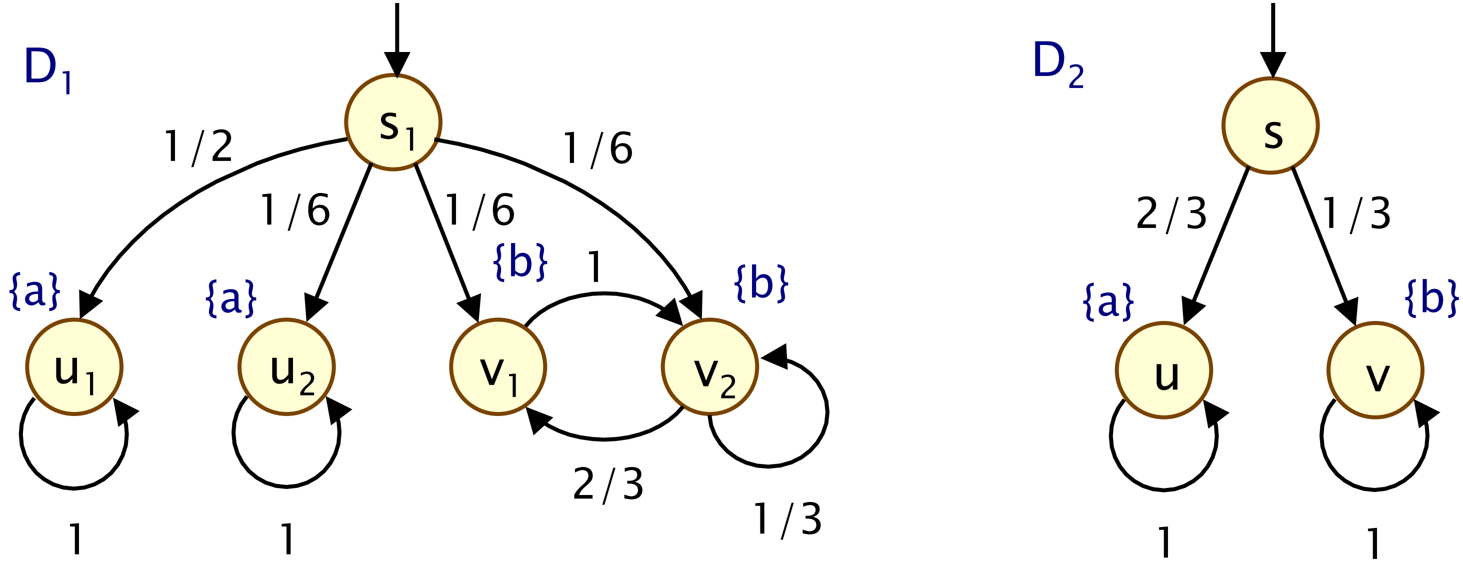
# Simple example

- Bisimulation relation ~
- Quotient of S under ~
  - denoted S/~
  - $\{ \{s_1\}, \{u_1, u_2\}, \{v_1, v_2\} \}$
- Bisimilar states:
  - $u_1 \sim u_2$
  - $v_1 \sim v_2$



# Bisimulation on DTMCs

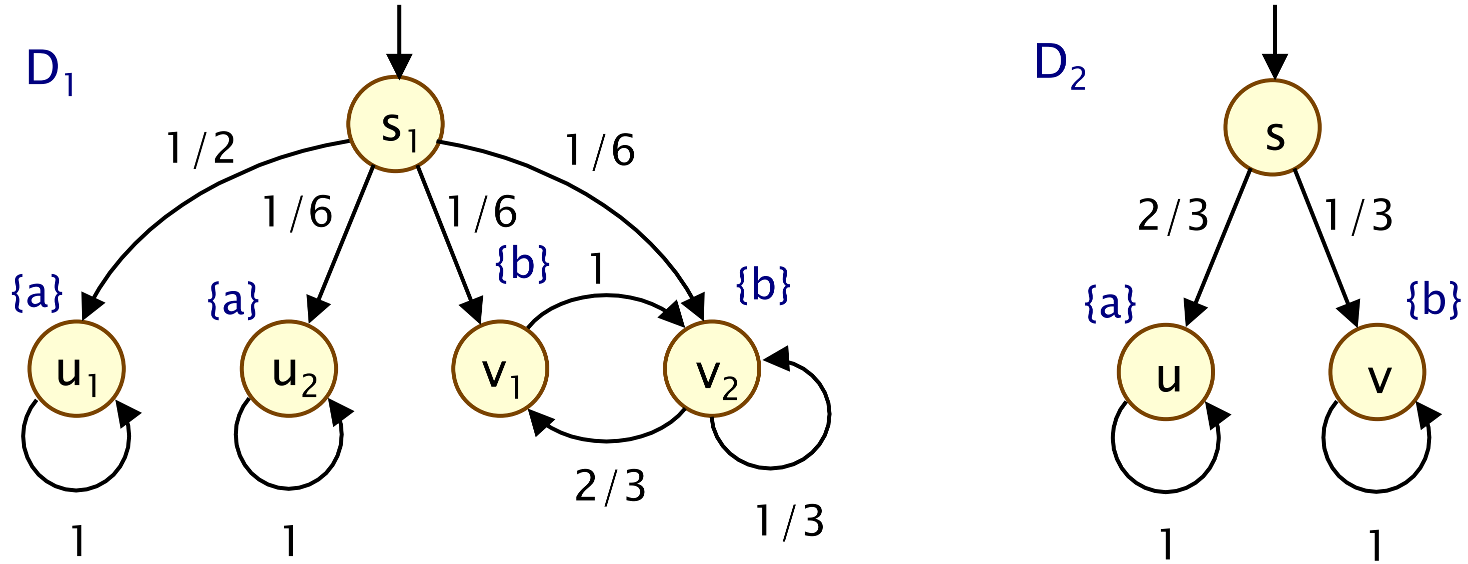
- Bisimulation between DTMCs $D_1$ and $D_2$
  - $D_1 \sim D_2$ if they have bisimilar initial states
- Formally:
  - state labellings for $D_1$ and $D_2$ over same set of atomic prop.s
  - bisimulation relation is over disjoint union of $D_1$ and $D_2$



# Simple example

- Bisimilar states:
  - $u_1 \sim u_2 \sim u$
  - $v_1 \sim v_2 \sim v$
  - $s_1 \sim s$

Bisimilar DTMCs: $D_1 \sim D_2$

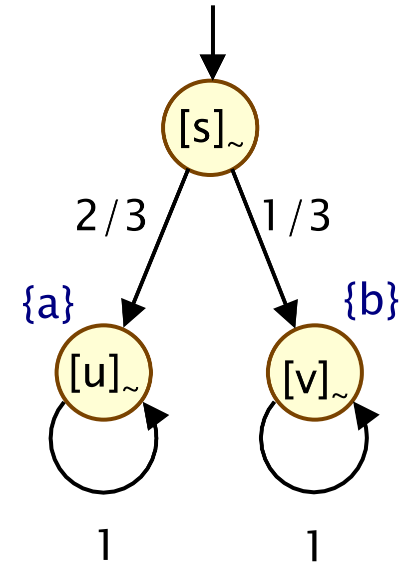


# Quotient DTMC

- For a DTMC $D = (S, s_{init}, P, L)$ and probabilistic bisimulation ~

- Quotient DTMC is
  - $D/\sim = (S', s'_{init}, P', L')$

- where:
  - $S' = S/\sim = \{ [s]_\sim \mid s \in S \}$
  - $s'_{init} = [s_{init}]_\sim$
  - $P'([s]_\sim, [s']_\sim) = P(s, [s']_\sim)$
  - $L'([s]_\sim) = L(s)$




well defined since bisimulation ensures $P(s, [s']_\sim)$ same for all s in $[s]_\sim$

# Bisimulation and PCTL

- Probabilistic bisimulation preserves all PCTL formulae

- For all states s and s':

$$s \sim s'$$
$$\Leftrightarrow$$
$$\text{for all PCTL formulae } \Phi,\ s \vDash \Phi \text{ if and only if } s' \vDash \Phi$$

- Note also:
  - every pair of non-bisimilar states can be distinguished with some PCTL formula
  - $\sim$ is the coarsest relation with this property
  - in fact, bisimulation also preserves all PCTL* formulae

# CTMC bisimulation

- Check equivalence of rates, not probabilities…

- An equivalence relation R on S is a probabilistic bisimulation on CTMC $C=(S,s_{init},\mathbf{R},L)$ if and only if for all $s_1$ R $s_2$:
  - $L(s_1) = L(s_2)$
  - $\mathbf{R}(s_1, T) = \mathbf{R}(s_2, T)$ for all classes T in S/R

- Alternatively, check:
  - $L(s_1) = L(s_2)$, $\mathbf{P}^{emb(C)}(s_1, T) = \mathbf{P}^{emb(C)}(s_2, T)$, $E(s_1) = E(s_2)$

- Bisimulation on CTMCs preserves CSL
  - (see [BHHK03] and also [DP03])

# Bisimulation minimisation

- More efficient to perform PCTL/CSL model checking on the quotient DTMC/CTMC
  - assuming quotient model can be constructed efficiently
  - (see [KKZJ07] for experimental results on this)

- Bisimulation minimisation
  - algorithm to construct quotient model
  - based on partition refinement
  - repeated splitting of an initially coarse partition
  - final partition is coarsest bisimulation wrt. initial partition
  - (optimisations/variants possible by changing initial partition)
  - complexity: $O(|\mathbf{P}| \cdot \log|S| + |AP| \cdot |S|)$ [DHS'03]
    - assuming suitable data structure used (splay trees)

# Bisimulation minimisation

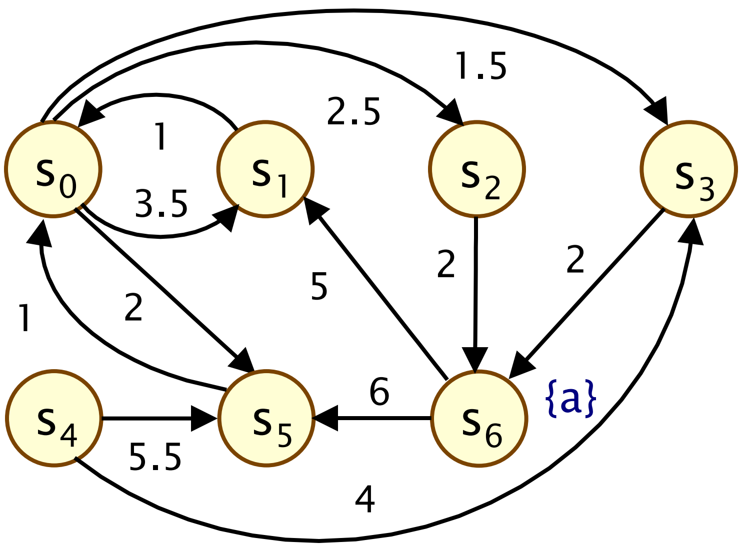
- 1. Start with initial partition
  - say $\Pi = \{ \{ s \in S \mid a \in L(s) \} \mid a \in AP \}$
- 2. Find a splitter $T \in \Pi$ for some block $B \in \Pi$
  - a splitter T is a block such that probability of going to T differs for some states in block B
  - i.e. $\exists s,s' \in B \,.\, \mathbf{P}(s,T) \neq \mathbf{P}(s',T)$

replace **P** with **R** for CTMCs

- 3. Split B into sub-blocks
  - such that $\mathbf{P}(s,T)$ is the same for all states in each sub-block
- 4. Repeat steps 2/3 until no more splitters exist
  - i.e. no change to partition $\Pi$

# CTMC example

- Consider model checking $P_{\sim p}$ [ $F^{[0,t]}$ a ] on this CTMC:




Minimisation:

$\Pi_0$: $B_1=\{s_0,s_1,s_2,s_3,s_4,s_5\}$, $B_2=\{s_6\}$

$B_2$ is a splitter for $B_1$

(since e.g. $\mathbf{R}(s_1,B_2)=0\neq 2=\mathbf{R}(s_2,B_2)$)

$\Pi_1$: $B_1=\{s_0,s_1,s_4,s_5\}$, $B_2=\{s_6\}$, $B_3=\{s_2,s_3\}$

$B_3$ is a splitter for $B_1$

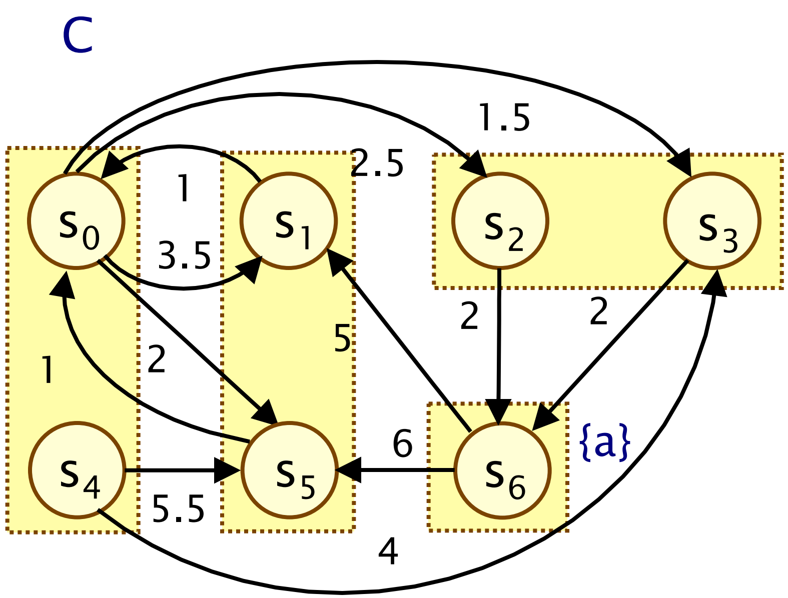
(since e.g. $\mathbf{R}(s_1,B_3)=0\neq 4=\mathbf{R}(s_0,B_3)$)

$\Pi_2$: $B_1=\{s_1,s_5\}$, $B_2=\{s_6\}$, $B_3=\{s_2,s_3\}$, $B_4=\{s_0,s_4\}$

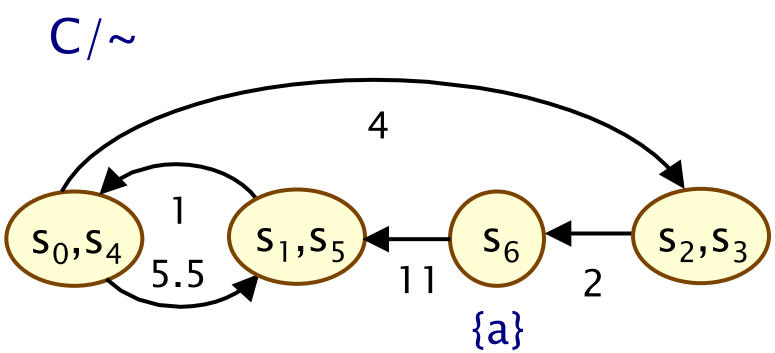
No more splitters…

$S/\sim\ = \{\ \{s_1,s_5\}, \{s_6\}, \{s_2,s_3\}, \{s_0,s_4\}\ \}$

# CTMC example...




$S/\sim \; = \{ \{s_1,s_5\}, \{s_6\}, \{s_2,s_3\}, \{s_0,s_4\} \}$




$$\mathrm{Prob}^{C}(s, F^{[0,t]}\, a) \;=\; \mathrm{Prob}^{C/\sim}(\{s_0,s_4\}, F^{[0,t]}\, a)$$

# Summary

- PRISM: Probabilistic model checker
  - for DTMCs, MDPs, CTMCs, …
  - high-level modelling language, property specifications
  - graphical user interface
- Counterexamples
  - essential ingredient of non-probabilistic model checking
  - for PCTL + DTMCs, need set of finite paths/evidences
  - computation: reduction to well-known graph problems
- Bisimulation
  - relates states/Markov chains with identical labelling and identical stepwise behaviour, preserves PCTL, CSL, …
  - minimisation: automated construction of quotient model

- Tomorrow: probabilistic timed automata (PTAs)