



# Probabilistic Model Checking with PRISM

Past, Present & Future

Marta Kwiatkowska



University  
of Oxford

Gethin Norman



University  
of Glasgow

Dave Parker



University of  
Birmingham

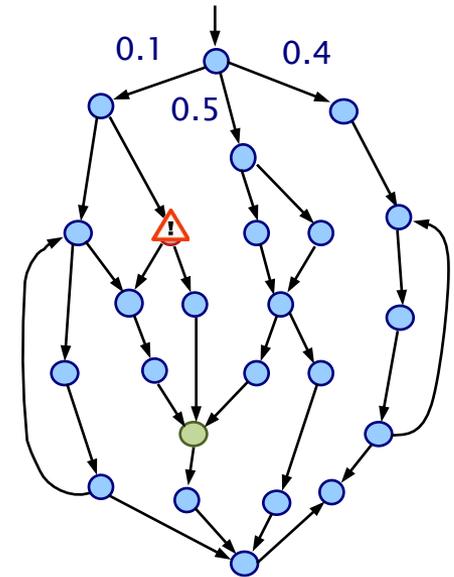
HVC 2016, Haifa, November 2016

# Outline

- Probabilistic model checking and PRISM
- Themes and trends
- Advances and applications
- Current research topics
- Challenges & future directions

# Probabilistic model checking

- Construction and analysis of probabilistic models
  - **probability**: failures, uncertainty, noise, randomisation, ...
  - **time**: delays, time-outs, failure rates, ...
  - **costs**: energy, resources, ...
- Quantitative correctness properties expressed in temporal logic, e.g.:
  - **trigger**  $\rightarrow P_{\geq 0.999} [ F^{\leq 20} \text{deploy} ]$
  - “the probability of the airbag deploying within 20 milliseconds of being triggered is at least 0.999”
  - reliability, timeliness, performance, efficiency, ...



# PRISM



- A (brief) history

- late 80s, early 90s: first underlying theory developed
- 2001: first official public release of PRISM
- 2011: version 4.0 - probabilistic real time systems
- 2013: PRISM-games – stochastic multi-player games

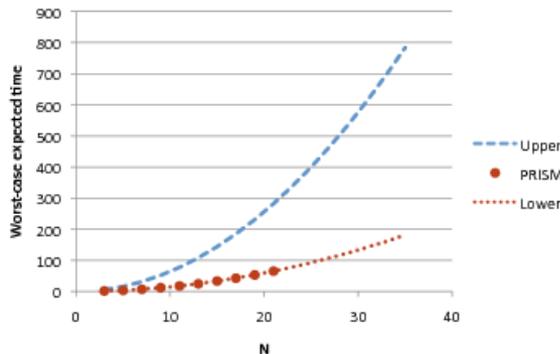
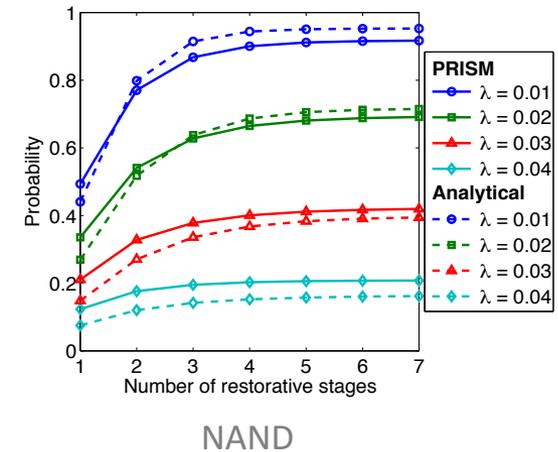
- PRISM today

- used in 100+ institutions; 50,000+ downloads
- broadly applicable; many diverse use cases
- many non-expert (and non-CS) users
- 300 external papers (no involvement from PRISM team)
- flaws found in real-systems; industrial usage

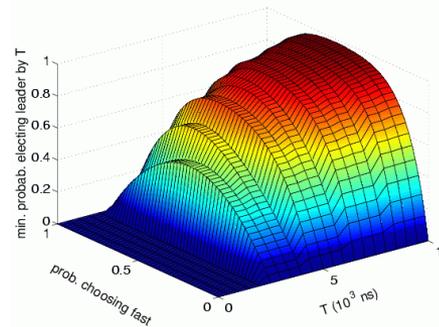


# What can we do with PRISM?

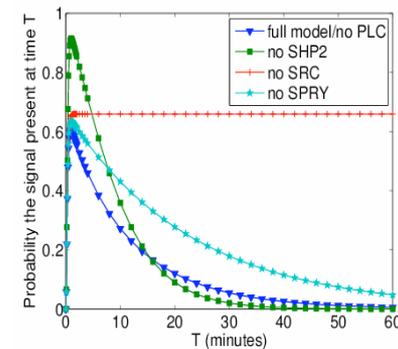
- Identify flaws in existing analyses
  - e.g. reliability of NAND multiplexing
- Investigate conjectures/models
  - e.g. Herman's self-stabilisation
  - e.g. FireWire root contention
  - e.g. cell signalling pathways (FGF)



Herman



FireWire



FGF

# Themes and trends

- Themes in the development of PRISM
  - **theory-to-practice** (and practice-to-theory)
  - wide **collaboration** (theory, algorithms, case studies)
  - **open source** software (and data)
  - overlaps/interacts with many **other disciplines**
- Trends
  - improvement in **scalability** to larger models
  - increasingly **expressive**/powerful classes of model
  - from verification problems to **control** problems
  - ever widening range of **application domains**

# Trends

## Models

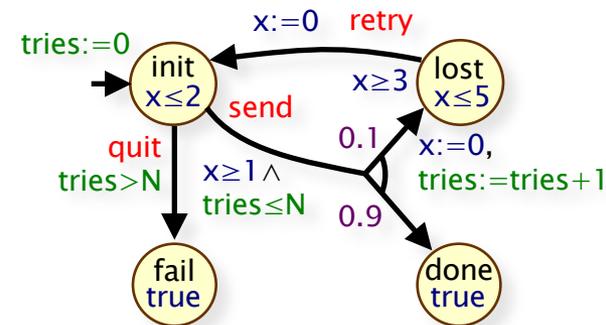
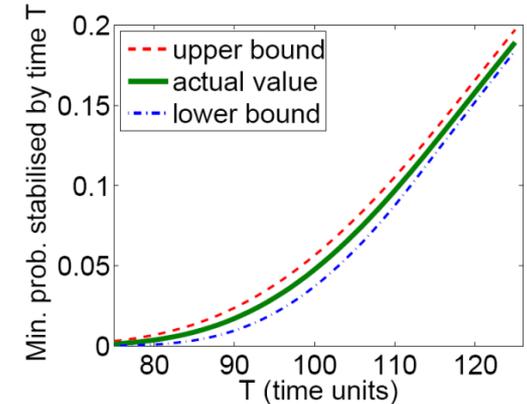
- 
- discrete-time Markov chains
  - probabilistic automata
  - continuous-time Markov chains
  - Markov decision processes
  - probabilistic timed automata
  - stochastic multi-player games
  - ...

## Application domains

- 
- randomised distributed algorithms
  - network/communication protocols
  - computer security
  - performance/reliability
  - systems biology
  - DNA computing
  - robotics & autonomous vehicles
  - wearable/implantable devices
  - ...

# Enabling technologies

- Symbolic model checking
  - [TACAS'00] [TACAS'02] [STTT'04] [CAV'06] ...
- Real-time probabilistic verification
  - [TCS'02] [FMSD'06] [Info&Comp'07] [FORMATS'09] ...
- Quantitative abstraction refinement
  - [QEST'06] [VMCAI'09] [FMSD'10] [QEST'11] ...
- Compositional verification
  - [TACAS'10] [QEST'10] [FASE'11] [Info&Comp'13] ...
- And more...
  - statistical model checking, symmetry reduction, bisimulation minimisation, ...

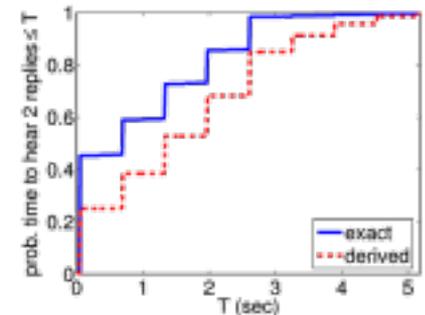


$$\frac{M_1 \models \langle A \rangle_{\geq q} \quad \langle A \rangle_{\geq q} M_2 \langle G \rangle_{\geq p}}{M_1 \parallel M_2 \models \langle G \rangle_{\geq p}}$$

# Case study: Bluetooth

- Device discovery between a pair of Bluetooth devices
  - performance essential for this phase
- Detailed model from official specification
  - two asynchronous 28-bit clocks
  - pseudo-random hopping between 32 frequencies
  - random waiting scheme to avoid collisions
  - 32 Markov chains, over  $3 \times 10^{10}$  states each
  - 17,179,869,184 initial configurations
- Symbolic probabilistic model checking
  - “worst-case expected discovery time is at most 5.17s”

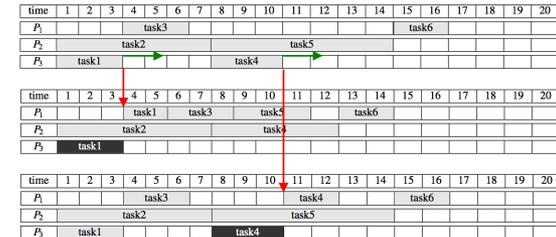
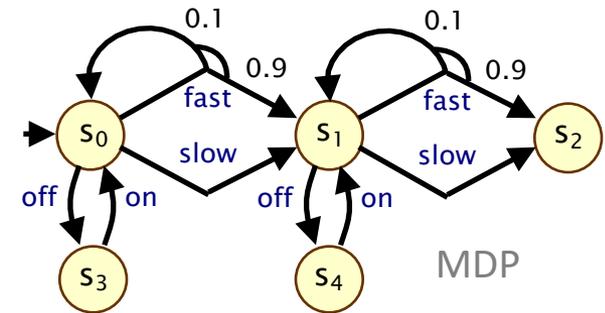
$$\text{freq} = [\text{CLK}_{16-12} + k + (\text{CLK}_{4-2,0} - \text{CLK}_{16-12}) \bmod 16] \bmod 32$$



# Strategy/controller synthesis

- Verification vs. control

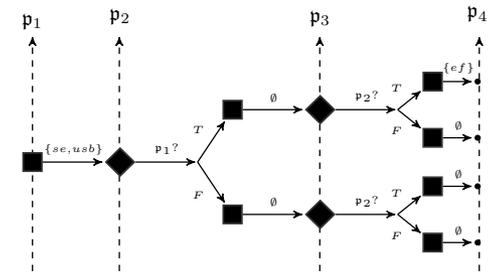
- **verify** that a system is “correct”, for any environment/adversary/... (counterexample yields flaw/attack/...)
- **synthesise** a “correct-by-construction” controller from formal specification (witness yields strategy/controller)



Task schedule [FMSD'13]

- Applications

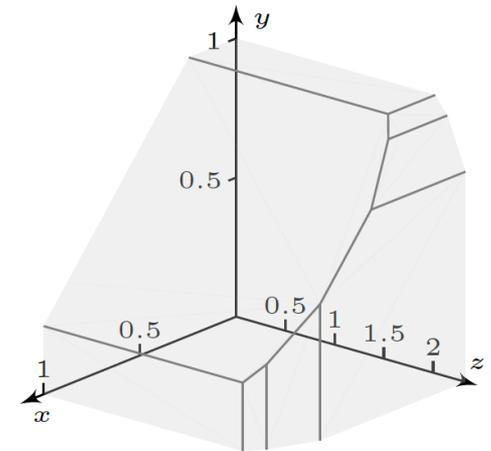
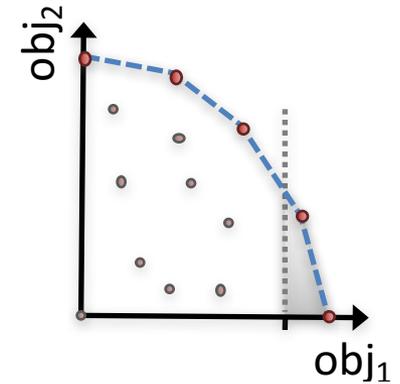
- dynamic power management, robots/autonomous vehicle navigation, task/network scheduling, security, ...



Attack-defence tree [CSF'16]

# Multiple objectives

- **Multi-objective controller synthesis** [LMCS'08] [TACAS'11]
  - trade-offs between conflicting objectives
- **Mix of optimisation and guarantees**
  - e.g. “what strategy **maximises** probability of message transmission, whilst **guaranteeing** expected battery life-time is  $> 10$  hrs?”
  - **Pareto curve** generation/approximation
- **Extensions**
  - permissive controller synthesis of multi-strategies for MDPs [LMCS'15]
  - multiple objectives for multi-player games (see later)



# Robots & autonomous systems

- Navigation for mobile service robots

- learnt probabilistic navigation maps
- LTL task specifications + controller synthesis
- ROS-based runtime planning implementation
- multi-objective probabilistic guarantees on task completion/duration [IROS'14/IJCAI'15/CDC'16]



- Autonomous underwater vehicle navigation

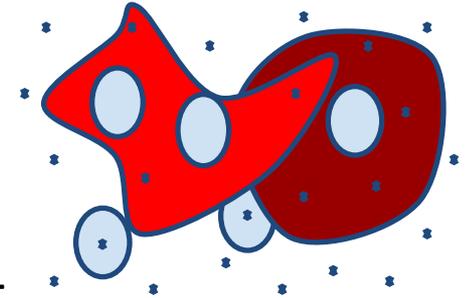
- incremental/parametric verification + controller synthesis
- probabilistic programming + machine learning to generate realistic component/environment models at runtime

# Parameter synthesis

- Synthesising models that are guaranteed to satisfy quantitative correctness properties is difficult
  - but we can synthesise **controllers** and **parameters**
- Parameter synthesis
  - given a **parametric** model and a property  $\phi$ ...
  - find the optimal parameter values, with respect to an objective function  $O$ , such that the property  $\phi$  is satisfied, if such values exist
- Quantitative parameter synthesis
  - **parameters**: timing delays, rates
  - **objectives**: optimise probability, reward/volume

# Quantitative parameter synthesis

- Timed/hybrid automata
  - find optimal **timing delays** [EMSOFT2014] [HSB'15] [HSCC'16]
  - constraint solving, discretisation + sampling
- Probabilistic timed automata
  - find delays to optimise **probability** [RP2014]
  - parametric symbolic abstraction-refinement
- Continuous-time Markov chains
  - find optimal **rates** [CMSB'14] [ActaInf'16], PRISM-PSY [TACAS'16]
  - constraint solving, uniformisation + sampling
- Focus: practical implementation, real-world usage





# Mobile autonomy challenge

- Autonomous systems
  - **interact** with their environment, which is possibly **adversarial**
  - have **goals/objectives**, which may **conflict**
  - take **decisions**
- Model as **stochastic games**
  - well known from, e.g., decision making in economics
  - many application domains: security, energy grid, etc
- Tool PRISM-games, extension of PRISM [TACAS'16]



# Stochastic multi-player games

- Probabilistic **temporal logic** with coalitions

- probabilities, rewards (reachability, total, mean-payoffs/ratios, ...)

[FMSD'13] [ICALP'16] [ECC'16]

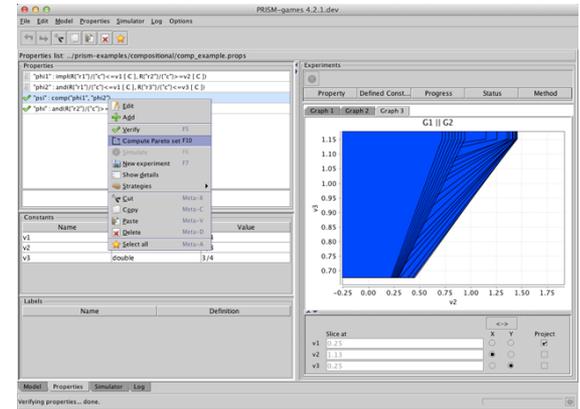
- **Multi-objective** strategy synthesis

- Pareto set computation and optimal achievable trade-offs [MFCS'13] [QEST'13] [TACAS'15]

- **Compositional** strategy synthesis [CONCUR'14] [Inf & Comp'16]

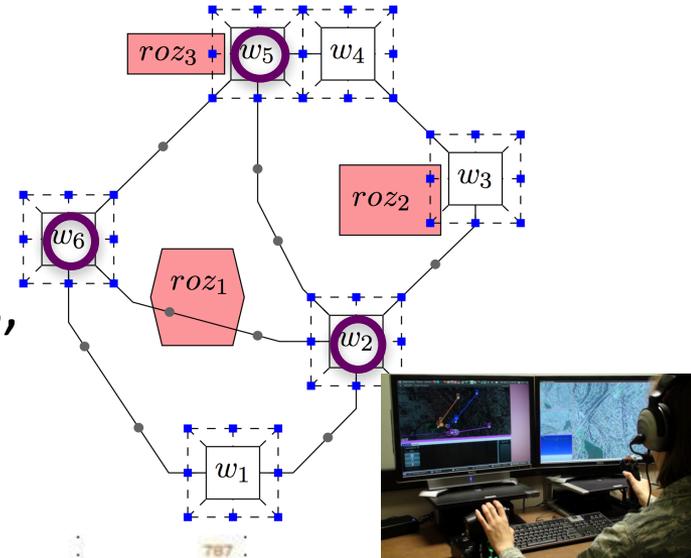
- assume-guarantee + multi-objective strategy synthesis

- e.g. local strategies for  $G_1 \models \phi_A$ ,  $G_2 \models \phi_A \Rightarrow \phi_B$   
compose to a global strategy for  $G_1 || G_2 \models \phi_B$

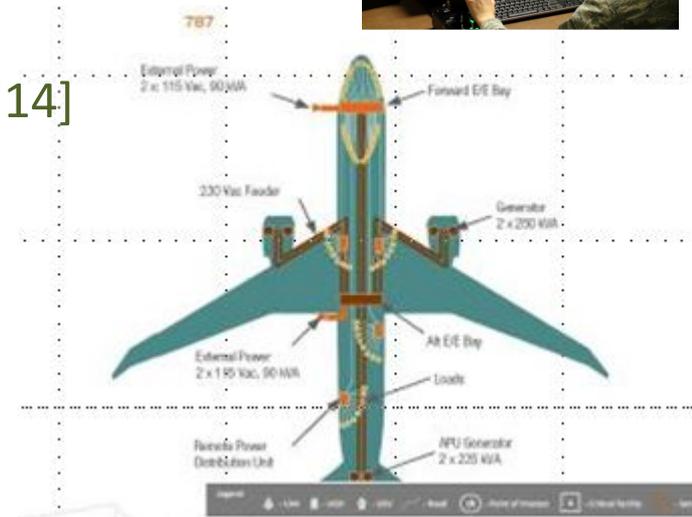


# Applications

- UAV path planning [ICCPs'15]
  - human operator + low-level piloting
  - **quantitative mission objectives**: minimise time/fuel, restricted zones, operator fatigue/workload
  - **multi-objective** MDPs, stoch. games



- Aircraft power distribution [CONCUR'14]
  - **compositional** strategy synthesis in stochastic games (PRISM-games)
  - specify control objectives in LTL using **mean payoff**



# Are games sufficient?

- Complex decisions!
  - goals
  - perception
  - situation awareness
  - context (social, regulatory)
- What about social subtleties?
- What to do in emergency?
  - **moral** decisions, **handover** to driver, obey traffic rules
- Need to make robots human-like...
  - need multi-modal communication, **cognitive reasoning**, trust, ethics, ...

Humans are pretty good at guessing what others on the road will do. Driverless cars are not—and that can be exploited.



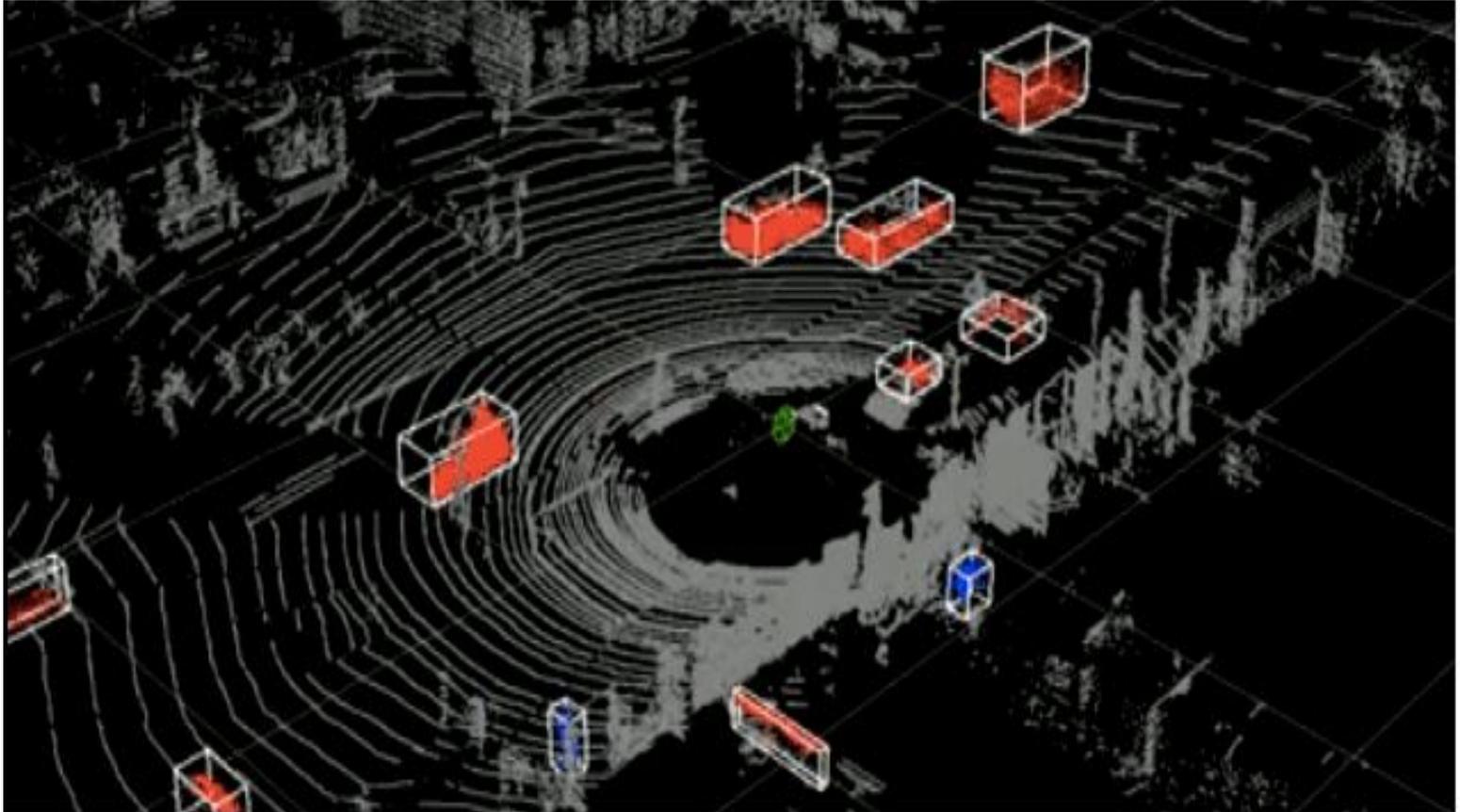
By Samuel English Anthony



# Quantitative verification for trust?

- Social trust: fundamental for mobile autonomy [LK16b]
  - influenced by **external** factors, such as social norms
  - also **internal**: personality, motivation, preferences
  - subjective: would you trust an autonomous taxi to take your child to school?
- Formulate a **temporal logic** to express X's trust in Y for G, based on probabilistic belief [HK17]
- Admits a **model checking** procedure, which can:
  - be used in **decision-making** for robots
  - **explain** decisions, i.e. who is accountable for the action

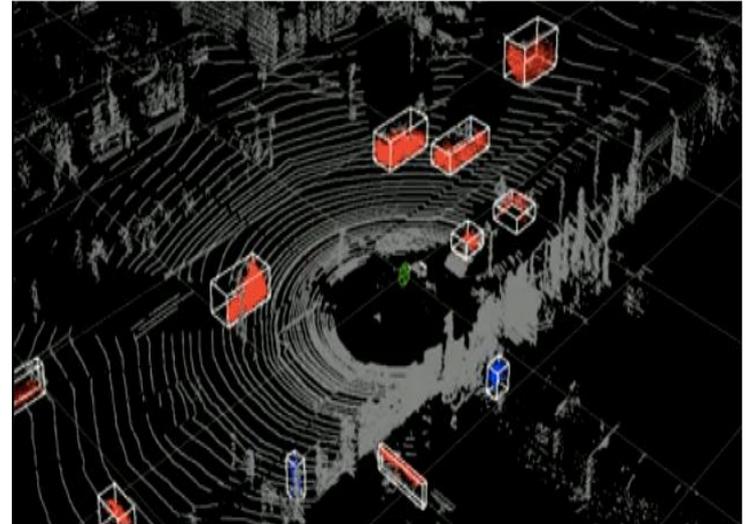
# Perception software



Credits: Oxford Robotics Institute

# Things that can go wrong...

- ...in **perception** software
  - sensor failure
  - object detection failure
- **Machine learning software**
  - not clear how it works
  - does **not** offer guarantees
- **Verification for machine learning?**
  - some progress towards safety verification for neural networks



# Personalisation challenge

- Device must **adapt** to physiology of human wearer
  - achieved through model parameterisation
  - parameter **estimation**, optimal parameter **synthesis**
- **Multiple uses**
  - **automation** of personalised medical intervention
  - device **safety** assurance, for testing
  - reproduce the unique characteristics for **authentication**
- **Focus on ECG based devices**
  - pacemaker models, heart models, synthetic ECGs
  - future work on anxiety monitoring and control

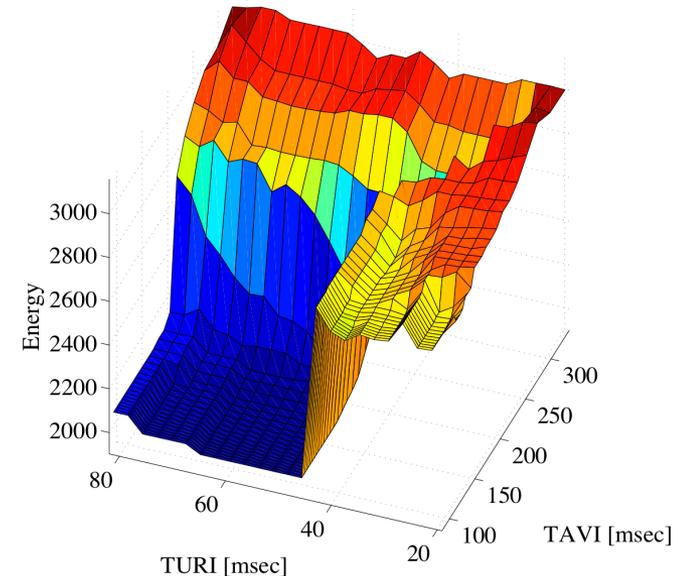
# Pacemaker verification/optimisation

- Hybrid model-based framework
  - timed automata model for pacemaker
  - hybrid heart models in Simulink (non-linear ODEs)



Copyright ©2008 Boston Scientific Corporation All rights reserved.

- Properties
  - (basic safety) maintain 60-100 beats per minute
  - optimisation of energy usage & cardiac output [HSB'16] [HSCC'16]
  - in-silico analysis of rate-adaptive pacemakers [ICHI'14]
  - hardware in the loop [EMBC'15]

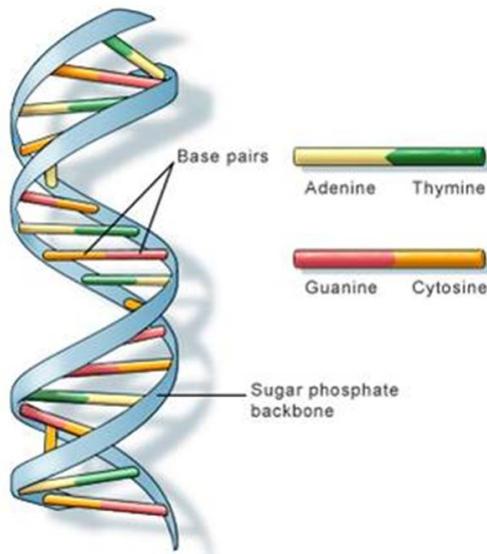


# DNA computation challenge

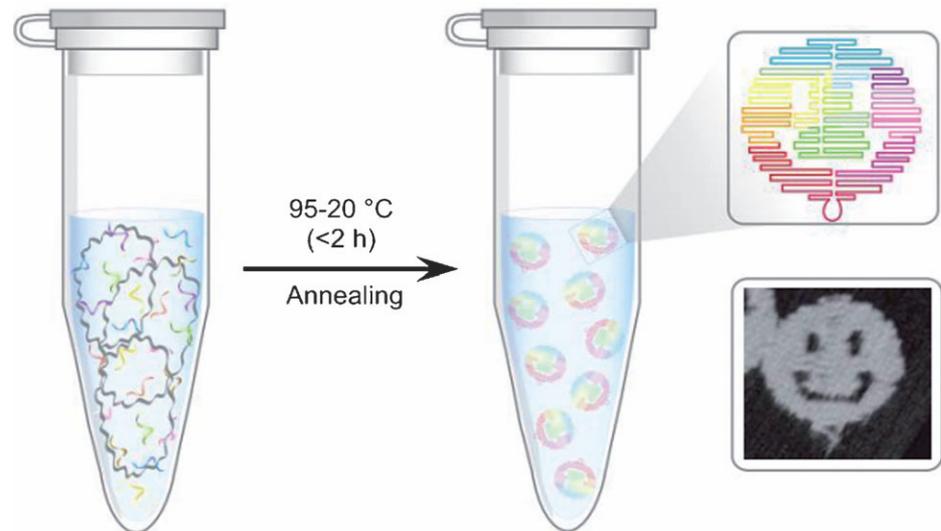
- Moore's law, hence need to make devices smaller...
- **DNA computation**, directly at the molecular level
  - DNA logic circuit designs & programmable nanorobotics
  - asynchronous DNA circuit designs [DNA'16]
- Many applications envisaged
  - e.g. bio-sensing, point of care diagnostics, ...
- Apply quantitative verification and synthesis to
  - find **design flaws** in DNA computing devices [JRSI'12]
  - analyse **reliability and performance** of molecular walkers
  - automatically **synthesise** reaction rates **to guarantee** a specified level of reliability

# DNA nanostructures

- **DNA origami** [Rothemund, *Nature* 2006]
  - DNA can self-assemble into structures – “**molecular IKEA?**”
  - **programmable** self-assembly (can form tiles, nanotubes, boxes that can open, etc.)



U.S. National Library of Medicine

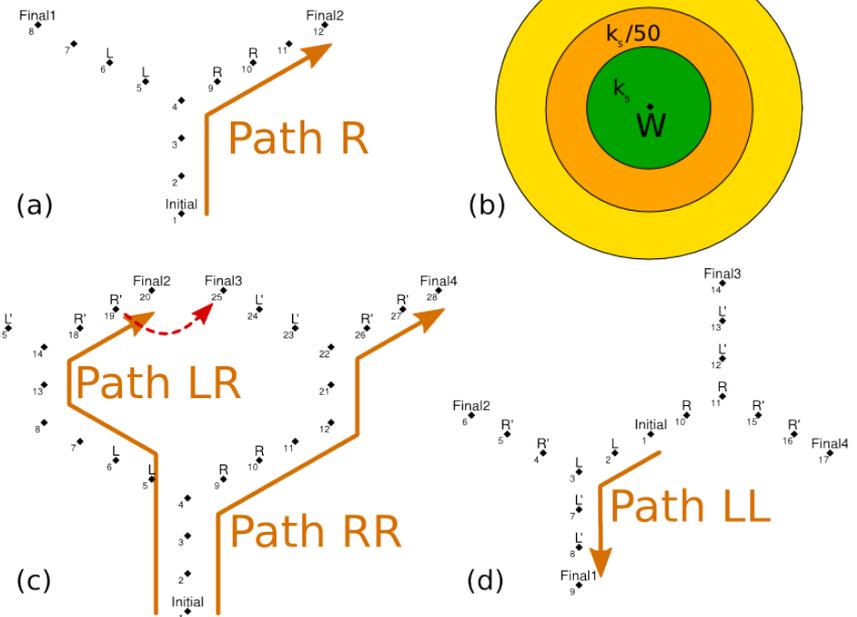


# DNA walker circuits

- Computing with DNA walkers [NatComp'14]

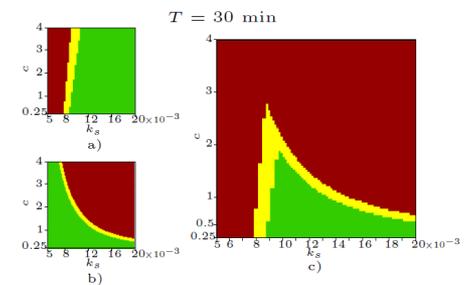
- branching tracks laid out on DNA origami tile
- starts at 'initial', signals when reaches 'final'
- can control 'left'/'right' decision
- any Boolean function

Decision circuits



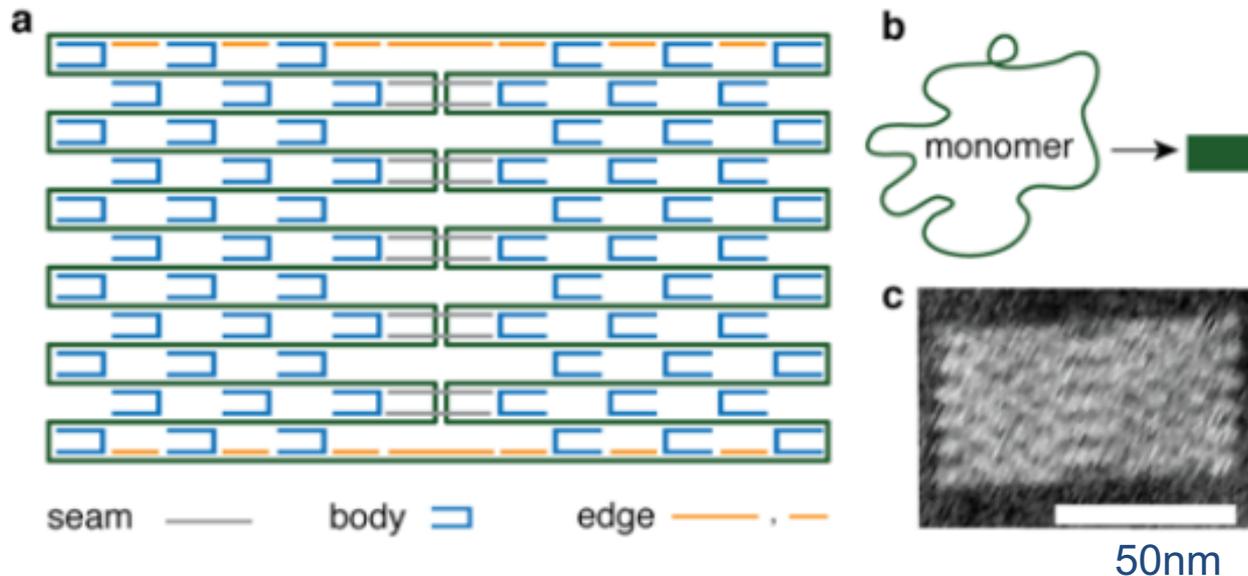
- Parameter synthesis of rates

- for guaranteed reliability level [CMSB'14]



# DNA origami tiles

- DNA origami tiles



- Aim: understand how to control the folding pathway
  - formulate an abstract **Markov chain** model
  - yields **predictions**; perform a range of experiments, consistent with predictions [Nature'15]

# Conclusions

- Probabilistic model checking & PRISM
  - 15 years since first official tool release
  - significant advances in underlying theory & technologies
  - successfully deployed in many application domains
- Many research challenges and applications ahead
  - verification, synthesis, learning, trust, cognitive models, ...
  - autonomous systems, DNA computing, personalised wearable/implantable devices, ...



<http://www.prismmodelchecker.org/>

# Acknowledgements

- **Contributors** (to PRISM & its underlying theory)

- Aistis Simaitis, Alberto Puggelli, Alessandro Bruni, Alexandru Mereacre, Alistair John Strachan, Andrej Tokarčik, Andrew Hinton, Antonio Pacheco, Archit Taneja, Ashutosh Trivedi, Benoit Barbot, Bruno Lacerda, Carlos Bederian, Charles Harley, Chris Thachuk, Christel Baier, Christian Dehnert, Christian von Essen, Christopher Ziegler, Chunyan Mu, Clemens Wiltsche, Dave Parker, Ernst Moritz Hahn, Frits Dannenberg, Fuzhi Wang, Ganindu Prabhashana, Gethin Norman, Håkan Younes, Holger Hermanns, Hongyang Qu, Jan Křetínský, Jens Katelaan, Jeremy Sproston, Joachim Klein, Joachim Meyer-Kayser, Joost-Pieter Katoen, Kenneth Chan, Klaus Draeger, Kousha Etessami, Lovejeet Singh, Lu Feng, Luca de Alfaro, Marcin Copik, Marco Diciolla, Maria Svorenova, Mark Kattenbelt, Markus Siegle, Marta Kwiatkowska, Mateusz Ujma, Maximilian Probst, Mihalis Yannakakis, Mike Arthur, Milan Ceska, Moshe Vardi, Muhammad Omer Saeed, Nick Hawes, Nicola Paoletti, Nicolas Basset, Nicolas Del Piano, Nishan Kamaleson, Paolo Ballarini, Pedro D'Argenio, Qixia Yuan, Radu Calinescu, Rashid Mehmood, Roberto Segala, Sebastian Vermehren, Sergio Giro, Steffen Märcker, Stephen Gilmore, Taolue Chen, Tingting Han, Vincent Nimal, Vojtěch Forejt, Xueyi Zou, Yi Zhang, Zak Cohen, ...

(and many more collaborators on case studies & projects)

- **Project funders**

