Lecture 6
PRISM

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Practicals

- 4 practical exercises
- 4 scheduled 2 hour practical sessions:
  - Tuesday 4-6pm, room 379, weeks 3, 4, 6 and 7
  - demonstrator: Aistis Simaitis

- Note:
  - you will also be expected to complete some of the practical work outside these hours
  - final assignment will include practical (PRISM) exercises

http://www.prismmodelchecker.org/courses/pmc1112/

Overview

- Tool support for probabilistic model checking
  - motivation, existing tools
- The PRISM model checker
  - functionality, features
  - modelling language
  - property specification
- Running example
  - leader election protocol
- PRISM tool demo

Motivation

- Complexity of PCTL model checking
  - generally polynomial in model size (number of states)
- State space explosion problem
  - models for realistic case studies are typically huge
- Clearly (efficient) tool support is required
- Benefits:
  - fully automated process
  - high-level languages/formalisms for building models
  - visualisation of quantitative results

Probabilistic model checkers

- PRISM (this lecture): DTMCs, MDPs, CTMCs, PTAs + rewards
- Markov chain model checkers
  - MRMC: DTMCs, CTMCs + reward extensions
  - PEPA toolset: CTMCs + CSL
- Markov decision process (MDP) tools
  - LiQuor: LTL verification for MDPs (Probmela language)
  - RAPTURE: prototype for abstraction/refinement of MDPs
  - ProbDiVinE: parallel/distributed LTL model checking of MDPs
- Simulation-based probabilistic model checking:
  - APMC, Ymer (both based on PRISM language), VESTA
- And more
  - APNN-Toolbox, SMART, CADP, Möbius, PASS, PARAM, …
  - see: http://www.prismmodelchecker.org/other-tools.php

The PRISM tool

- PRISM: Probabilistic symbolic model checker
  - developed at Birmingham/Oxford University, since 1999
  - free, open source (GPL)
  - versions for Linux, Unix, Mac OS X, Windows, 64-bit OSs

- Modelling of:
  - DTMCs, CTMCs, MDPs + costs/rewards
  - probabilistic timed automata (PTAs) (not covered here)

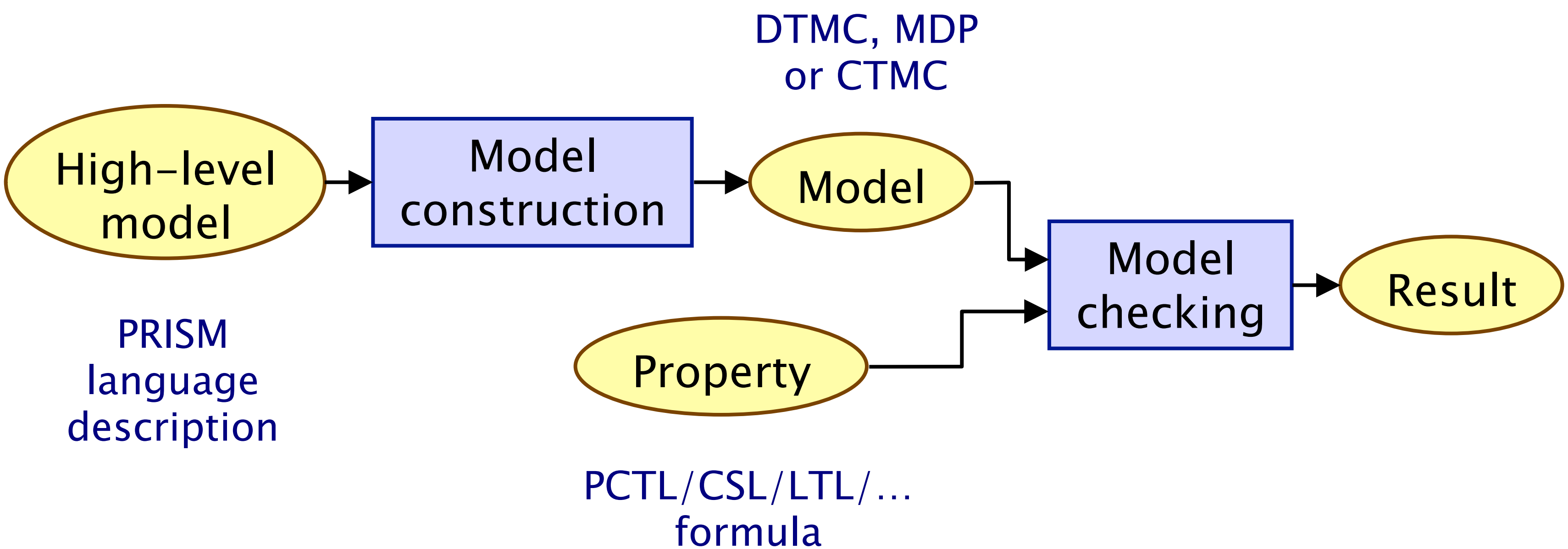
- Model checking of:
  - PCTL, CSL, LTL, PCTL* + extensions + costs/rewards

PRISM functionality

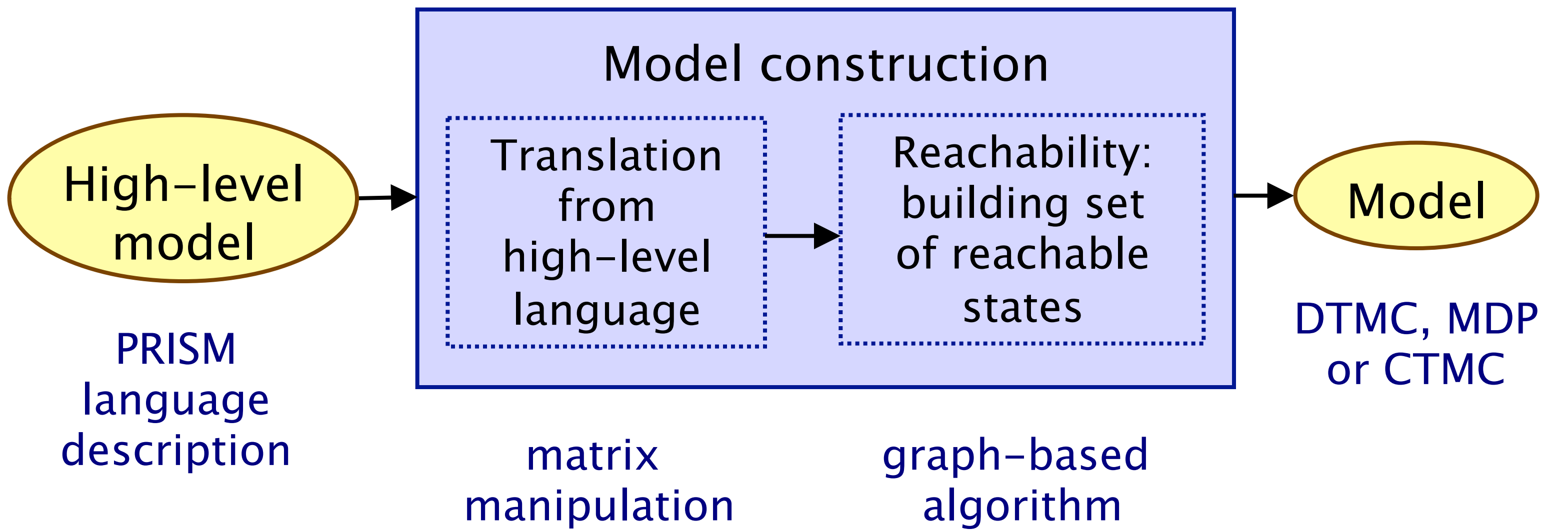
- High-level modelling language
- Wide range of model analysis methods
  - efficient symbolic implementation techniques
  - also: approximate verification using simulation + sampling
- Graphical user interface
  - model/property editor
  - discrete-event simulator - model traces for debugging, etc.
  - easy automation of verification experiments
  - graphical visualisation of results
- Command-line version
  - same underlying verification engines
  - useful for scripting, batch jobs

Probabilistic model checking

- Overview of the probabilistic model checking process
  - two distinct phases: model construction, model checking



Model construction



Modelling languages/formalisms

- Many high-level modelling languages, formalisms available
- For example:
  - probabilistic/stochastic process algebras
  - stochastic Petri nets
  - stochastic activity networks
- Custom languages for tools, e.g.:
  - PRISM modelling language
  - Probmela (probabilistic variant of Promela, the input language for the model checker SPIN) – used in LiQuor

PRISM modelling language

- Simple, textual, state-based language
  - modelling of DTMCs, CTMCs, MDPs, …
  - based on Reactive Modules [AH99]

- Basic components…
- Modules:
  - components of system being modelled
  - composed in parallel
- Variables
  - finite (integer ranges or Booleans)
  - local or global
  - all variables public: anyone can read, only owner can modify

PRISM modelling language

- Guarded commands
  - describe behaviour of each module
  - i.e. the changes in state that can occur
  - labelled with probabilities (or, for CTMCs, rates)
  - (optional) action labels

$$\underbrace{[\mathit{send}]}_{\text{action}}\ \underbrace{(s=2)}_{\text{guard}}\ \text{->}\ \underbrace{p_{loss}}_{\text{probability}} : \underbrace{(s'=3)\&(\mathit{lost}'=\mathit{lost}+1)}_{\text{update}}\ +\ \underbrace{(1-p_{loss})}_{\text{probability}} : \underbrace{(s'=4);}_{\text{update}}$$

PRISM modelling language

- Parallel composition
  - model multiple components that can execute independently
  - for DTMC models, mostly assume components operate synchronously, i.e. move in lock-step
- Synchronisation
  - simultaneous transitions in more than one module
  - guarded commands with matching action-labels
  - probability of combined transition is product of individual probabilities for each component
- More complex parallel compositions can be defined
  - using process-algebraic operators
  - other types of parallel composition, action hiding/renaming

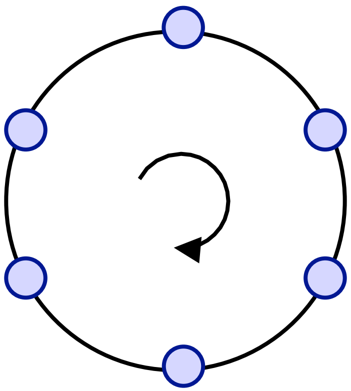
Simple example

```
module M1
    x : [0..3] init 0;
    [a] x=0 -> (x'=1);
    [b] x=1 -> 0.5:(x'=2) + 0.5:(x'=3);
endmodule
```

```
module M2
    y : [0..3] init 0;
    [a] y=0 -> (y'=1);
    [b] y=1 -> 0.4:(y'=2) + 0.6:(y'=3);
endmodule
```

Example: Leader election

- Randomised leader election protocol
  - due to Itai & Rodeh (1990)
- Set-up: N nodes, connected in a ring
  - communication is synchronous (lock-step)
- Aim: elect a leader
  - i.e. one uniquely designated node
  - by passing messages around the ring
- Protocol operates in rounds. In each round:
  - each node choose a (uniformly) random id $\in \{0,\ldots,k-1\}$
  - (k is a parameter of the protocol)
  - all nodes pass their id around the ring
  - if there is (maximum) unique id, node with this id is the leader
  - if not, try again with a new round



PRISM code

PRISM property specifications

- Based on (probabilistic extensions of) temporal logic
  - incorporates PCTL, CSL, LTL, PCTL*
  - also includes: quantitative extensions, costs/rewards
- Leader election properties
  - $P_{\geq 1}$ [F elected]
    - with probability 1, a leader is eventually elected
  - $P_{>0.8}$ [$F^{\leq k}$ elected]
    - with probability greater than 0.8, a leader is elected within k steps
- Usually focus on quantitative properties:
  - $P_{=?}$ [$F^{\leq k}$ elected]
    - what is the probability that a leader is elected within k steps?

PRISM property specifications

- Best/worst-case scenarios
  - combining "quantitative" and "exhaustive" aspects

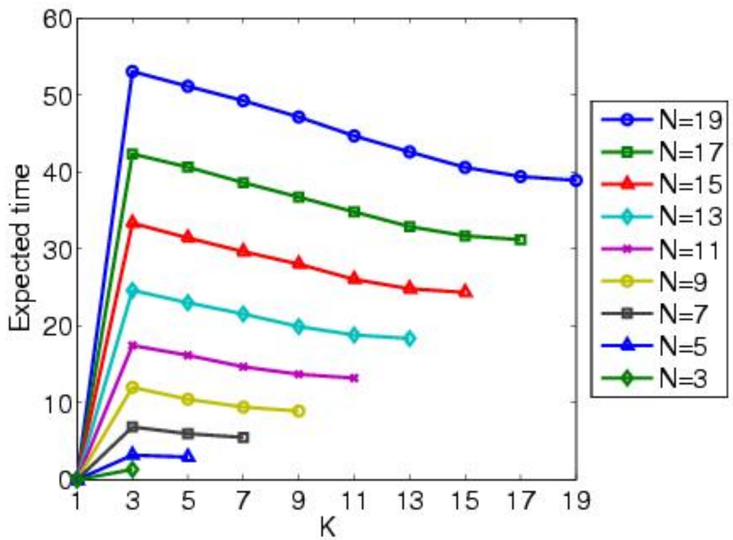
- e.g. computing values for a range of states…

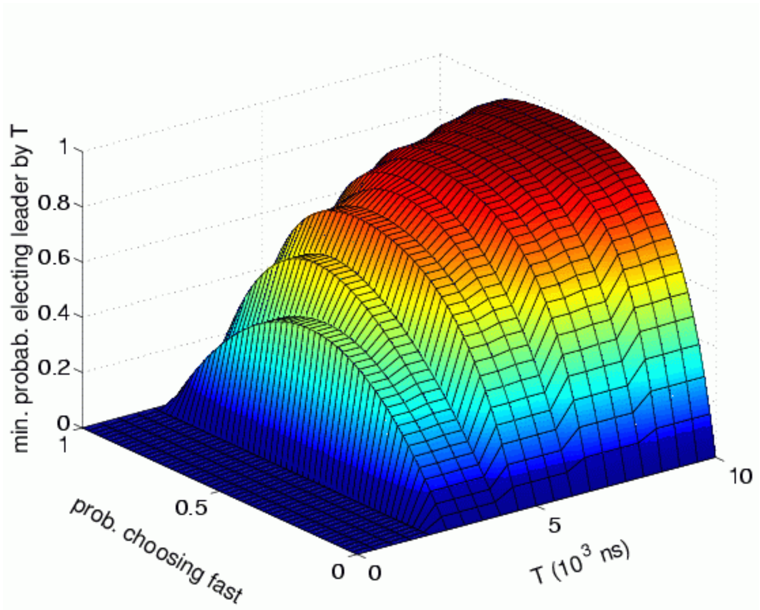
- $P_{=?}$ [$F^{\leq t}$ elected {tokens≤k}{min}] –
  - "minimum probability of the leader election algorithm completing within t steps from any state where there are at most k tokens"

- $R_{=?}$ [F end {"init"}{max}] –
  - "maximum expected run-time over all possible initial configurations"

PRISM property specifications

- Experiments:
  - ranges of model/property parameters
  - e.g. $P_{=?}$ [$F^{\leq T}$ error] for N=1..5, T=1..100
    where N is some model parameter and T a time bound
  - identify patterns, trends, anomalies in quantitative results





PRISM…

More info on PRISM

- PRISM website: http://www.prismmodelchecker.org/
  - tool download: binaries, source code (GPL)
  - on-line example repository (50+ case studies)
  - on-line documentation:
    - PRISM manual
    - PRISM tutorial
  - support: help forum, bug tracking, feature requests
  - related publications, talks, tutorials, links
- Course practicals info at:
  - http://www.prismmodelchecker.org/courses/pmc1112/