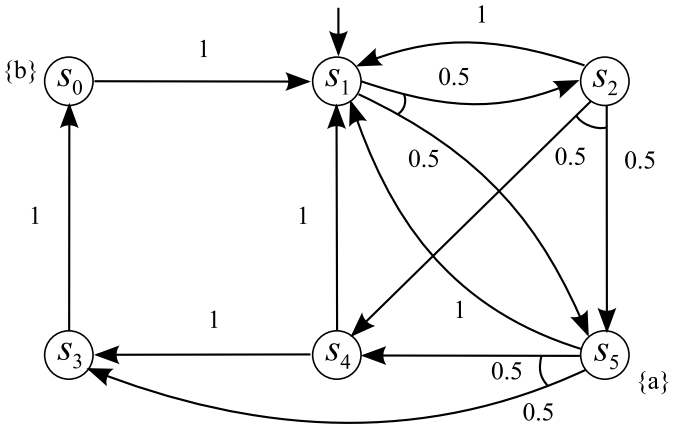
Problem Sheet 2: MDPs, automata and LTL

1. This question concerns the following MDP:



   (a) Execute the PCTL model checking algorithm to determine which states of the MDP satisfy the following PCTL formula:

      i. $\mathrm{P}_{\leq 0.3}[\,\neg a \,\mathrm{U}\, b\,]$

   (b) For the PCTL property $\mathrm{P}_{\leq 0.3}[\,\neg a \,\mathrm{U}\, b\,]$ above, deduce a memoryless adversary which results in the computed probabilities and give the corresponding DTMC.

   (c) On the DTMC from (b), give a counterexample for $\mathrm{P}_{\leq 0.3}[\,\neg a \,\mathrm{U}\, b\,]$ in state s_2.

   (d) Identify all sets of states in the MDP that can form end components. For each the following PCTL* formula, list the states that satisfy it, justifying your answer.

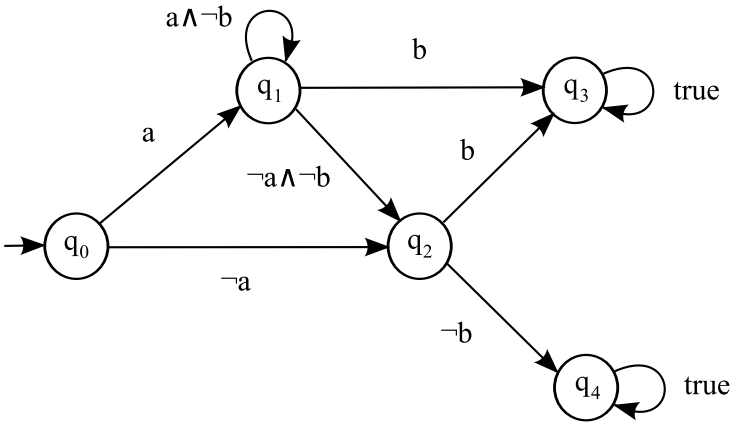
      i. $\mathrm{P}_{\geq 1}[\,\mathrm{GF}\, a\,]$

2. Show whether each of the two PCTL formulae below is *satisfiable*. That is, either provide an example of an MDP for which at least one state satisfies the formula, or prove that this is impossible.

   (a) $\mathrm{P}_{>0.5}[\,\mathrm{X}\, a\,] \wedge \mathrm{P}_{<0.5}[\,\mathrm{F}\, a\,]$

   (b) $\neg\mathrm{P}_{\leq 0.6}[\,\mathrm{X}\, a\,] \wedge \neg\mathrm{P}_{\geq 0.4}[\,\mathrm{F}\, a\,]$

3. Below is a deterministic Rabin automaton. The acceptance condition comprises the single pair $(\{q_4\}, \{q_3\})$.



(a) Give an equivalent LTL formula for this automaton.

(b) Give a small MDP on which the minimum and maximum probabilities of satisfying this formula differ.