Problem Sheet 1: DTMCs, PCTL and model checking
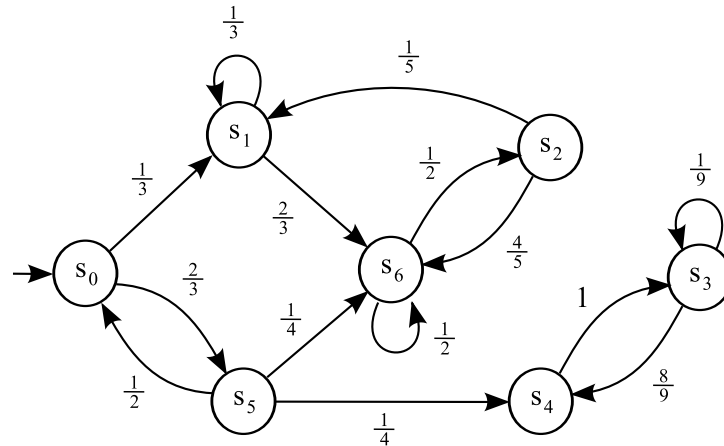
1. Consider the DTMC below:

Let $A = \{s_3\}$ and $B = \{s_2\}$.

   (a) Compute the probability measure of the *union* of the following cylinder sets:
       $\mathrm{Cyl}(s_0 s_1)$, $\mathrm{Cyl}(s_0 s_5 s_6)$, $\mathrm{Cyl}(s_0 s_5 s_4 s_3)$, $\mathrm{Cyl}(s_0 s_1 s_6)$

   (b) Compute the probability, from each state of the Markov chain, of reaching a state in A.

   (c) What is the probability, from the initial state, of reaching the set of states $A \cup B$?

   (d) What is the probability, from the initial state, that a state from $A \cup B$ is visited infinitely often?

2. Recall from lectures that, for a DTMC, the PCTL formula $\mathrm{P}_{>0}[\,\mathrm{F}\,a\,]$ is equivalent to the CTL formula $\mathrm{EF}\,a$, where a is an atomic proposition. For each of the following pairs of PCTL and CTL formulae, either show that the two formulae are equivalent or provide a simple finite DTMC which illustrates the difference.

   (a) PCTL: $\mathrm{P}_{\geq 1}[\,\mathrm{G}\,a\,]$ CTL: $\mathrm{AG}\,a$

   (b) PCTL: $\mathrm{P}_{>0}[\,\mathrm{G}\,a\,]$ CTL: $\mathrm{EG}\,a$

3. Consider the DTMC below. Illustrate the execution of the PCTL model checking algorithms to determine which states of the Markov chain satisfy:

   (a) $\mathrm{P}_{\geq \frac{17}{19}}[\,b\,\mathrm{U}\,c\,]$

   (b) $\mathrm{P}_{\geq \frac{1}{2}}[\,\mathrm{X}\,\mathrm{P}_{>\frac{1}{3}}[\,(b \vee c)\,\mathrm{U}^{\leq 2}\,(b \wedge c)\,]\,]$