Probabilistic model checking with PRISM

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Lecture plan

- Course slides and lab session
  - http://www.prismmodelchecker.org/courses/imt16/
- 3 sessions: lectures 9-11
  - 1 – Discrete time Markov chains (DTMCs)
  - 2 – Markov decision processes (MDPs)
  - 3 – LTL model checking for DTMCs/MDPs
- For extended versions of this material
  - and an accompanying list of references
  - see: http://www.prismmodelchecker.org/lectures/

Probabilistic models

	Fully probabilistic	Nondeterministic
Discrete time	Discrete-time Markov chains (DTMCs)	Markov decision processes (MDPs)
		Simple stochastic games (SMGs)
Continuous time	Continuous-time Markov chains (CTMCs)	Probabilistic timed automata (PTAs)
		Interactive Markov chains (IMCs)

Part 3

LTL Model Checking

Overview (Part 3)

- Linear temporal logic (LTL)
- Strongly connected components
- ω-automata (Büchi, Rabin)
- LTL model checking for DTMCs
- LTL model checking for MDPs
- New developments and beyond PRISM

Limitations of PCTL

- PCTL, although useful in practice, has limited expressivity
  - essentially: probability of reaching states in X, passing only through states in Y (and within k time-steps)

- One useful approach: extend models with costs/rewards
  - see slides for the last two lectures

- Another direction: Use more expressive logics. e.g.:
  - LTL [Pnu77] – (non-probabilistic) linear-time temporal logic
  - PCTL* [ASB+95,BdA95] – which subsumes both PCTL and LTL
  - both allow path operators to be combined
  - (in PCTL, $P_{\sim p}$ […] always contains a single temporal operator)

LTL – Linear temporal logic

- LTL syntax (path formulae only)
  - $\psi ::= \text{true} \mid a \mid \psi \wedge \psi \mid \neg\psi \mid X\,\psi \mid \psi\ U\ \psi$
  - where $a \in AP$ is an atomic proposition
  - usual equivalences hold: $F\,\phi \equiv \text{true}\ U\ \phi$, $G\,\phi \equiv \neg(F\,\neg\phi)$

- LTL semantics (for a path ω)
  - $\omega \vDash \text{true}$ always
  - $\omega \vDash a \quad\Leftrightarrow\quad a \in L(\omega(0))$
  - $\omega \vDash \psi_1 \wedge \psi_2 \quad\Leftrightarrow\quad \omega \vDash \psi_1$ and $\omega \vDash \psi_2$
  - $\omega \vDash \neg\psi \quad\Leftrightarrow\quad \omega \nvDash \psi$
  - $\omega \vDash X\,\psi \quad\Leftrightarrow\quad \omega[1\ldots] \vDash \psi$
  - $\omega \vDash \psi_1\ U\ \psi_2 \quad\Leftrightarrow\quad \exists k \geq 0 \text{ s.t. } \omega[k\ldots] \vDash \psi_2 \wedge \forall i<k\ \omega[i\ldots] \vDash \psi_1$

where $\omega(i)$ is i^{th} state of ω, and $\omega[i\ldots]$ is suffix starting at $\omega(i)$

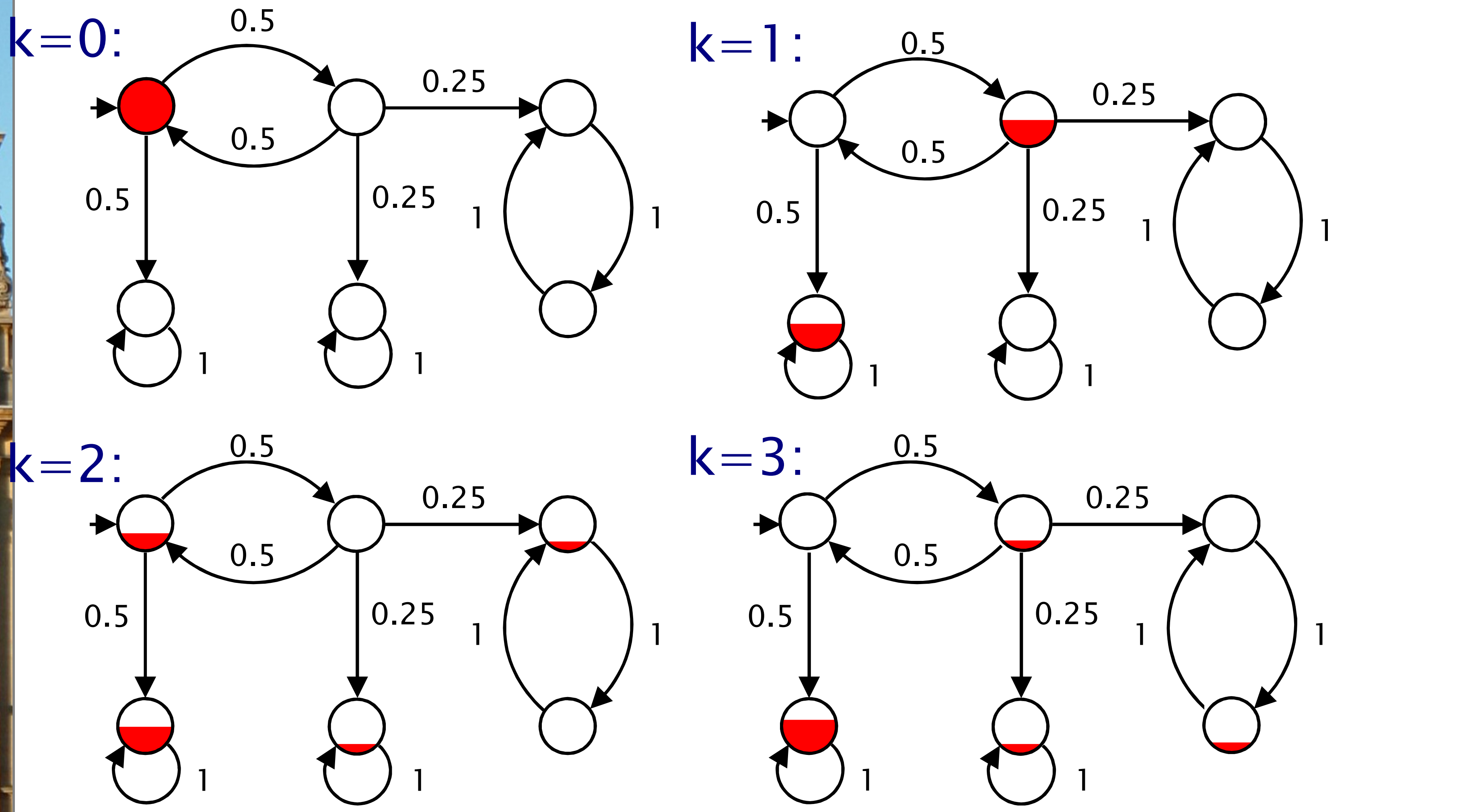
LTL examples

- $(\mathrm{F}\ tmp\_fail_1) \land (\mathrm{F}\ tmp\_fail_2)$
  - "both servers suffer temporary failures at some point"
- GF ready
  - "the server always eventually returns to a ready-state"
- FG error
  - "an irrecoverable error occurs"
- $\mathrm{G}\ (req \rightarrow \mathrm{X}\ ack)$
  - "requests are always immediately acknowledged"

LTL for DTMCs

- Same idea as PCTL: probabilities of sets of path formulae
  - for a state s of a DTMC and an LTL formula ψ:
  - $\text{Prob}(s, \psi) = \Pr_s \{ \omega \in \text{Path}(s) \mid \omega \models \psi \}$
  - all such path sets are measurable [Var85]
- A (probabilistic) LTL specification often comprises an LTL (path) formula and a probability bound
  - e.g. $P_{\geq 1}$ [GF ready] – "with probability 1, the server always eventually returns to a ready-state"
  - e.g. $P_{\leq 0.01}$ [FG error] – "with probability at most 0.01, an irrecoverable error occurs"
- PCTL* subsumes both LTL and PCTL
  - e.g. $P_{>0.5}$ [GF $crit_1$] $\wedge$ $P_{>0.5}$ [GF $crit_2$]

Long-run behaviour of DTMCs



Strongly connected components

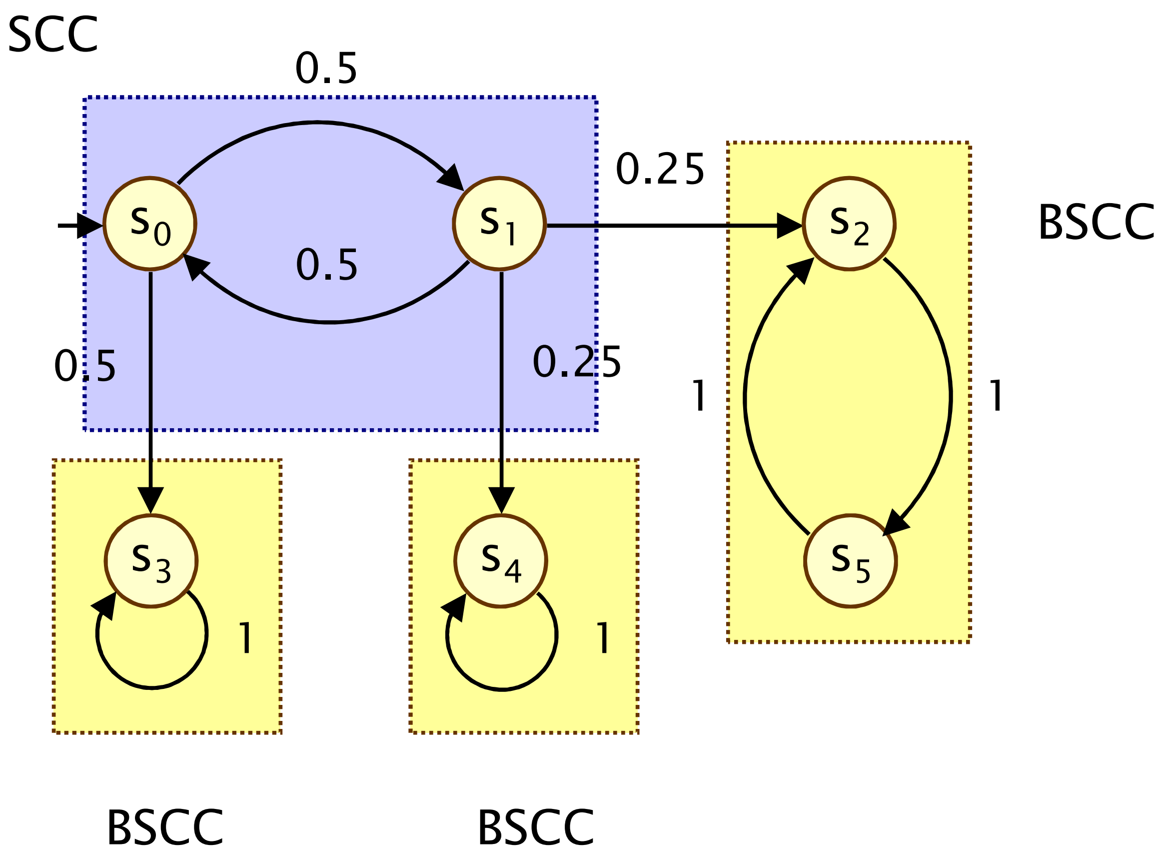
- Long-run properties of DTMCs rely on an analysis of their underlying graph structure (i.e. ignoring probabilities)

- Strongly connected set of states T
  - for any pair of states s and s' in T, there is a path from s to s', passing only through states in T

- Strongly connected component (SCC)
  - a maximally strongly connected set of states (i.e. no superset of it is also strongly connected)

- Bottom strongly connected component (BSCC)
  - an SCC T from which no state outside T is reachable from T

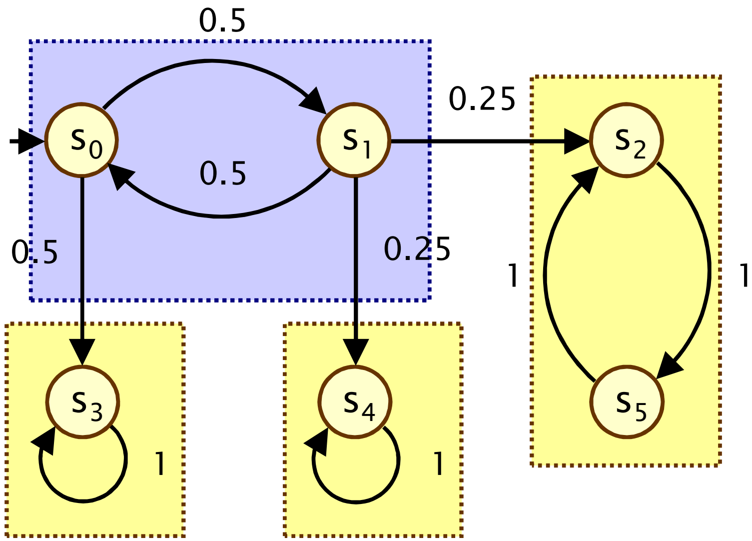
Example – (B)SCCs



Fundamental property of DTMCs

- Fundamental property of (finite) DTMCs…

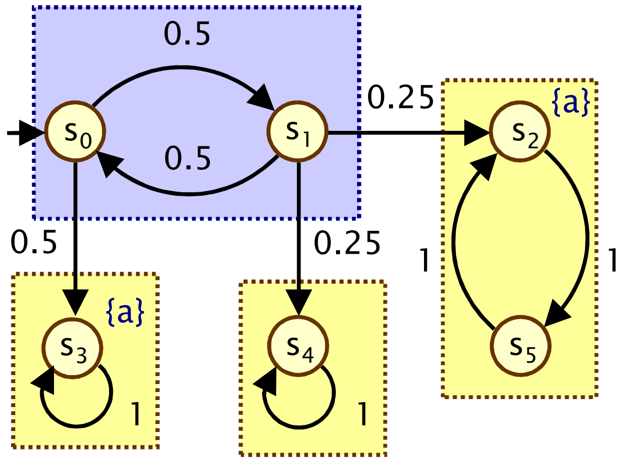
- With probability 1, some BSCC will be reached and all of its states visited infinitely often



- Formally:
  - Pr_{s0} ($s_0 s_1 s_2 \ldots$ | $\exists$ $i \geq 0$, $\exists$ BSCC T such that
    $\forall$ $j \geq i$ $s_j \in$ T and
    $\forall$ $s \in T$ $s_k = s$ for infinitely many k) = 1

LTL model checking for DTMCs

- LTL model checking for DTMCs relies on:
  - computing the probability Prob(s, ψ) for LTL formula ψ
  - reduces to probability of reaching a set of "accepting" BSCCs
  - 2 simple cases: GF a and FG a…



- Prob(s, GF a) = Prob(s, F T_{GFa})
  - where T_{GFa} = union of all BSCCs containing some state satisfying a

- Prob(s, FG a) = Prob(s, F T_{FGa})
  - where T_{FGa} = union of all BSCCs containing only a-states

- To extend this idea to arbitrary LTL formula, we use ω-automata…

Example:

Prob(s_0, GF a)

= Prob(s_0, F T_{GFa})

= Prob(s_0, F {s_3,s_2,s_5})

= 2/3 + 1/6 = 5/6

Overview (Part 3)

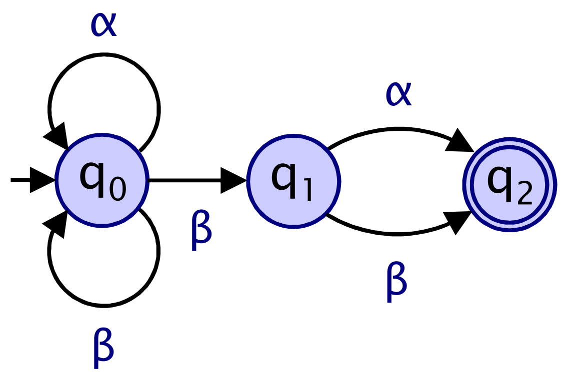
- Linear temporal logic (LTL)
- Strongly connected components
- ω-automata (Büchi, Rabin)
- LTL model checking for DTMCs
- LTL model checking for MDPs
- New developments and beyond PRISM

Reminder – Finite automata

- A regular language over alphabet Σ
  - is a set of finite words $L \subseteq \Sigma^*$ such that either:
  - $L = L(E)$ for some regular expression E
  - $L = L(A)$ for some nondeterministic finite automaton (NFA) A
  - $L = L(A)$ for some deterministic finite automaton (DFA) A

- Example:

  Regexp: $(\alpha+\beta)^*\beta(\alpha+\beta)$ NFA A:



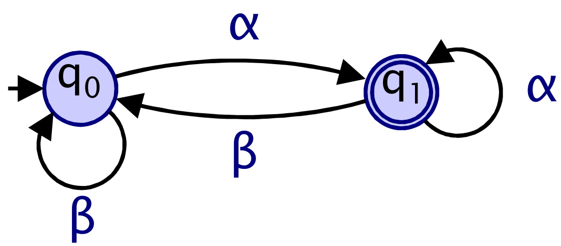
- NFAs and DFAs have the same expressive power
  - we can always determinise an NFA to an equivalent DFA
  - (with a possibly exponential blow-up in size)

Büchi automata

- ω-automata represent sets of infinite words $L \subseteq \Sigma^\omega$
  - e.g. Büchi automata, Rabin automata, Streett, Muller, …

- A nondeterministic Büchi automaton (NBA) is…
  - a tuple $A = (Q, \Sigma, \delta, Q_0, F)$ where:
  - Q is a finite set of states
  - Σ is an alphabet
  - $\delta : Q \times \Sigma \rightarrow 2^Q$ is a transition function
  - $Q_0 \subseteq Q$ is a set of initial states
  - $F \subseteq Q$ is a set of "accept" states

Example:
words $w \in \{\alpha,\beta\}^\omega$
with infinitely many α



- NBA acceptance condition
  - language $L(A)$ for A contains $w \in \Sigma^\omega$ if there is a corresponding run in A that passes through states in F infinitely often

ω-regular properties

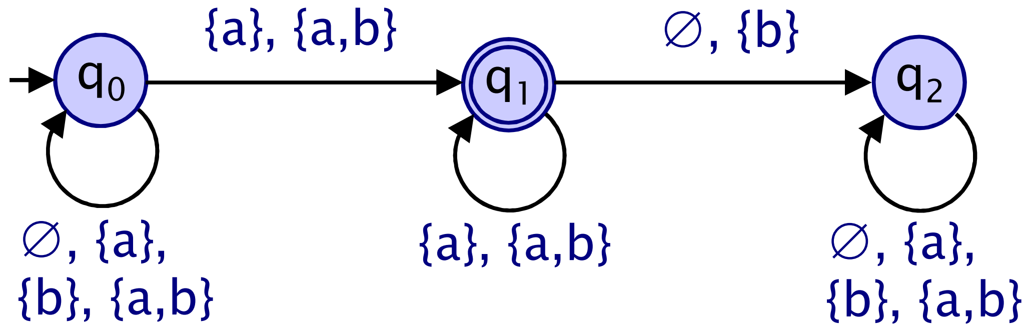
- Consider a model, i.e. an LTS/DTMC/MDP/…
  - for example: DTMC $D = (S, s_{init}, P, Lab)$
  - where labelling Lab uses atomic propositions from set AP

- We can capture properties of these using ω-automata
  - let $\omega \in Path(s)$ be some infinite path in D
  - $trace(\omega) \in (2^{AP})^\omega$ denotes the projection of state labels of ω
  - i.e. $trace(s_0 s_1 s_2 s_3 \ldots) = Lab(s_0)Lab(s_1)Lab(s_2)Lab(s_3)\ldots$
  - can specify a set of paths of D with an ω-automaton over 2^{AP}

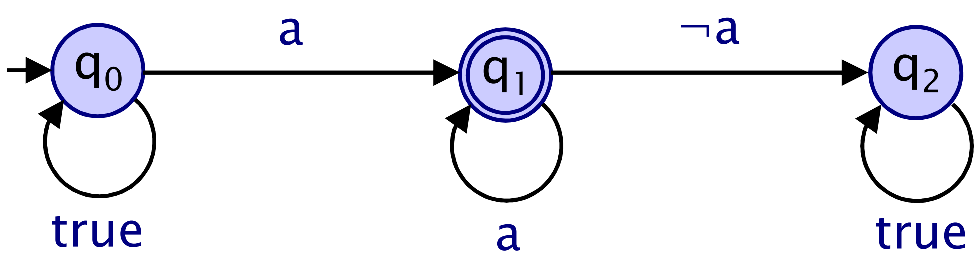
- Let $Prob^D(s, A)$ denote the probability…
  - from state s in a discrete-time Markov chain D
  - of satisfying the property specified by automaton A
  - i.e. $Prob^D(s, A) = Pr^D_s\{\ \omega \in Path(s) \mid trace(\omega) \in L(A)\ \}$

Example

- Nondeterministic Büchi automaton
  - for LTL formula FG a, i.e. “eventually always a”
  - for a DTMC with atomic propositions AP = {a,b}



- We abbreviate this to just:



Büchi automata + LTL

- Nondeterministic Büchi automata (NBAs)
  - define the set of ω-regular languages

- ω-regular languages are more expressive than LTL
  - can convert any LTL formula ψ over atomic propositions AP
  - into an equivalent NBA A_ψ over 2^{AP}
  - i.e. $\omega \vDash \psi \Leftrightarrow trace(\omega) \in L(A_\psi)$ for any path ω
  - for LTL-to-NBA translation, see e.g. [VW94], [DGV99], [BK08]
  - worst-case: exponential blow-up from $|\psi|$ to $|A_\psi|$

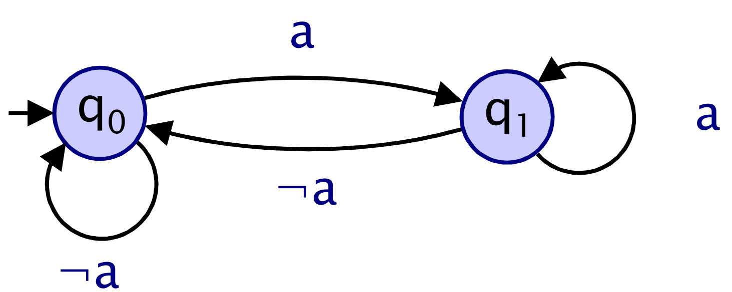
- But deterministic Büchi automata (DBAs) are less expressive
  - e.g. there is no DBA for the LTL formula FG a
  - for probabilistic model checking, need deterministic automata
  - so we use deterministic Rabin automata (DRAs)

Deterministic Rabin automata

- A deterministic Rabin automaton is a tuple $(Q, \Sigma, \delta, q_0, Acc)$:
  - Q is a finite set of states, $q_0 \in Q$ is an initial state
  - Σ is an alphabet, $\delta : Q \times \Sigma \rightarrow Q$ is a transition function
  - $Acc = \{ (L_i, K_i) \}_{i=1..k} \subseteq 2^Q \times 2^Q$ is an acceptance condition

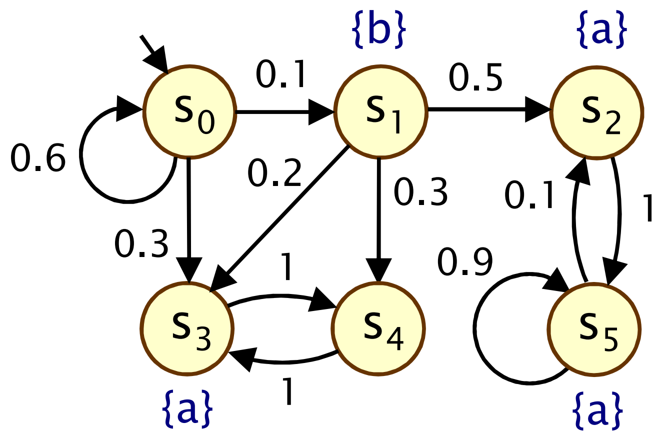
- A run of a word on a DRA is accepting iff:
  - for some pair (L_i, K_i), the states in L_i are visited finitely often and (some of) the states in K_i are visited infinitely often
  - or in LTL: $\bigvee_{1 \leq i \leq k} (\text{FG}\, \neg L_i \wedge \text{GF}\, K_i)$

- Example: DRA for FG a
  - acceptance condition is $Acc = \{ (\{q_0\},\{q_1\}) \}$



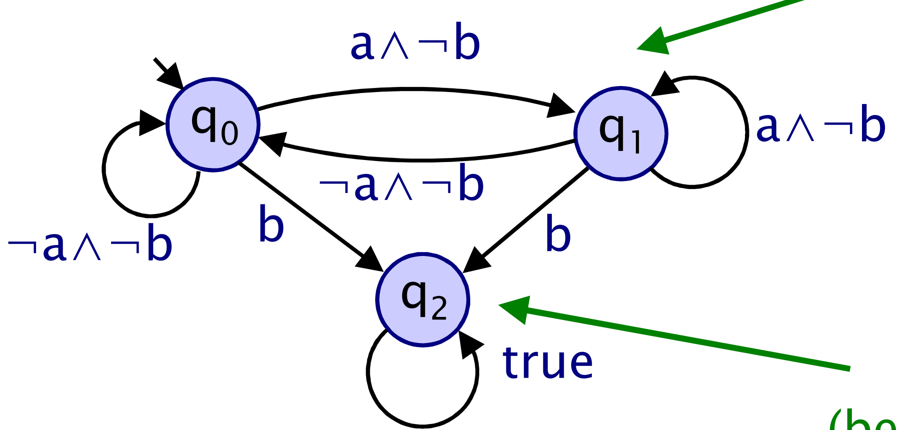
LTL model checking for DTMCs

- LTL model checking for DTMC D and LTL formula ψ
- 1. Construct DRA A_ψ for ψ
- 2. Construct product D ⊗ A of DTMC D and DRA A_ψ
- 3. Compute $Prob^D(s, \psi)$ from DTMC D ⊗ A
- Running example:
  - compute probability of satisfying LTL formula $\psi = G\neg b \wedge GF\ a$ on:



Example – DRA

- DRA A_ψ for $\psi = G\neg b \wedge GF\ a$
  - acceptance condition is Acc = { ({},{q_1}) }
  - (i.e. this is actually a deterministic Büchi automaton)



Need to visit here infinitely often to satisfy GF a

If G¬b violated (because we see a b), end up stuck here

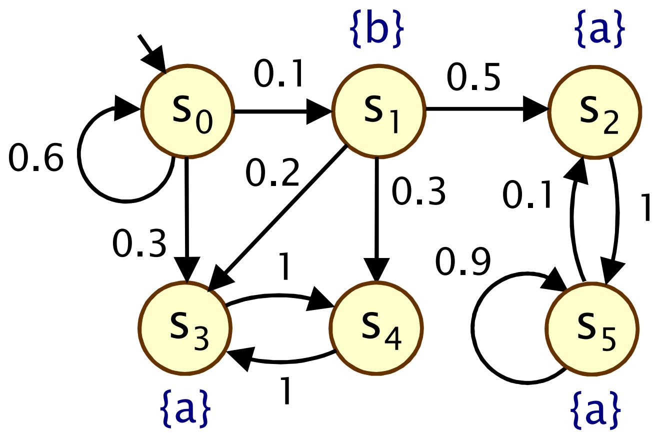
Product DTMC for a DRA

- We construct the product DTMC
  - for DTMC D and DRA A, denoted $D \otimes A$
  - $D \otimes A$ can be seen as an unfolding of D with states (s,q), where q records state of automaton A for path fragment so far
  - since A is deterministic, $D \otimes A$ is a also a DTMC
  - each path in D has a corresponding (unique) path in $D \otimes A$
  - the probabilities of paths in D are preserved in $D \otimes A$

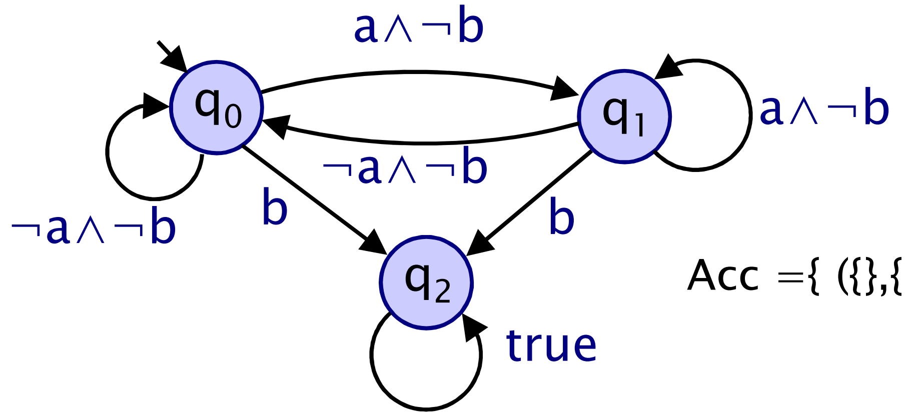
- Formally, for $D = (S, s_{init}, P, L)$ and $A = (Q, \Sigma, \delta, q_0, \{(L_i, K_i)\}_{i=1..k})$
  - $D \otimes A$ is the DTMC $(S \times Q, (s_{init}, q_{init}), P', L')$ where:
  - $q_{init} = \delta(q_0, L(s_{init}))$
  - $P'((s_1,q_1),(s_2,q_2)) = \begin{cases} P(s_1,s_2) & \text{if } q_2 = \delta(q_1, L(s_2)) \\ 0 & \text{otherwise} \end{cases}$
  - $l_i \in L'(s,q)$ if $q \in L_i$ and $k_i \in L'(s,q)$ if $q \in K_i$

Example – Product DTMC

DTMC D

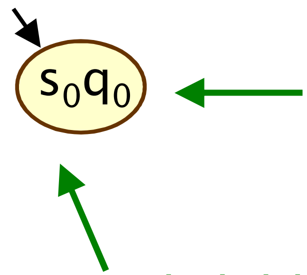


DRA A_ψ for $\psi = G\neg b \wedge GF\ a$



Acc ={ ({},{q_1}) }

Product DTMC $D \otimes A_\psi$

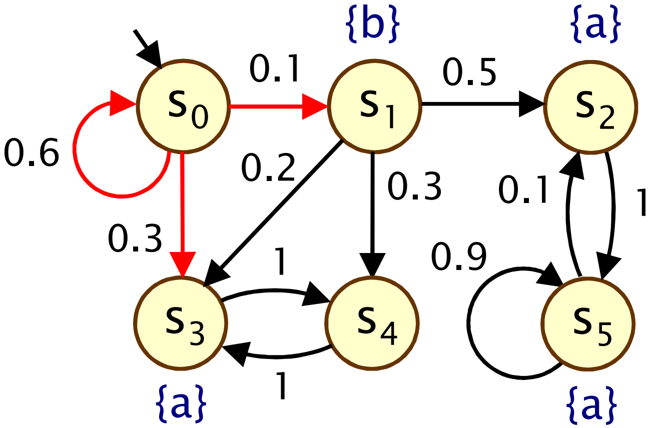


s_0 satisfies neither a nor b so we stay in q_0 in DRA A_ψ

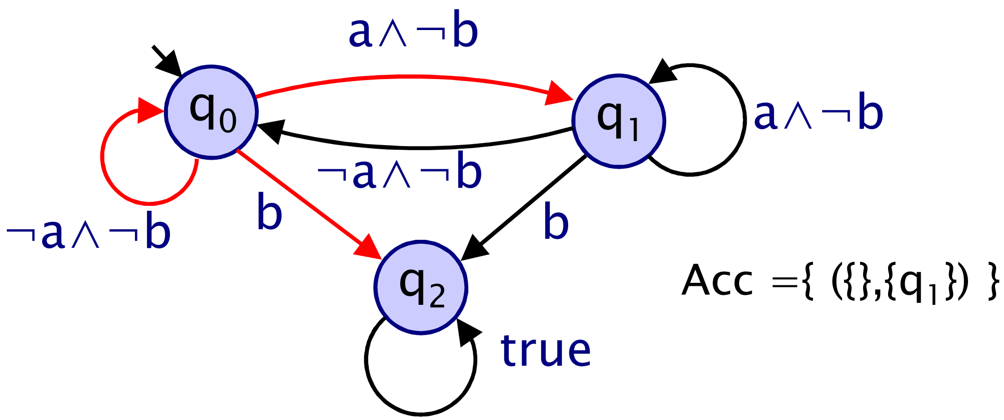
s_0 is initial state of DTMC D

Example – Product DTMC

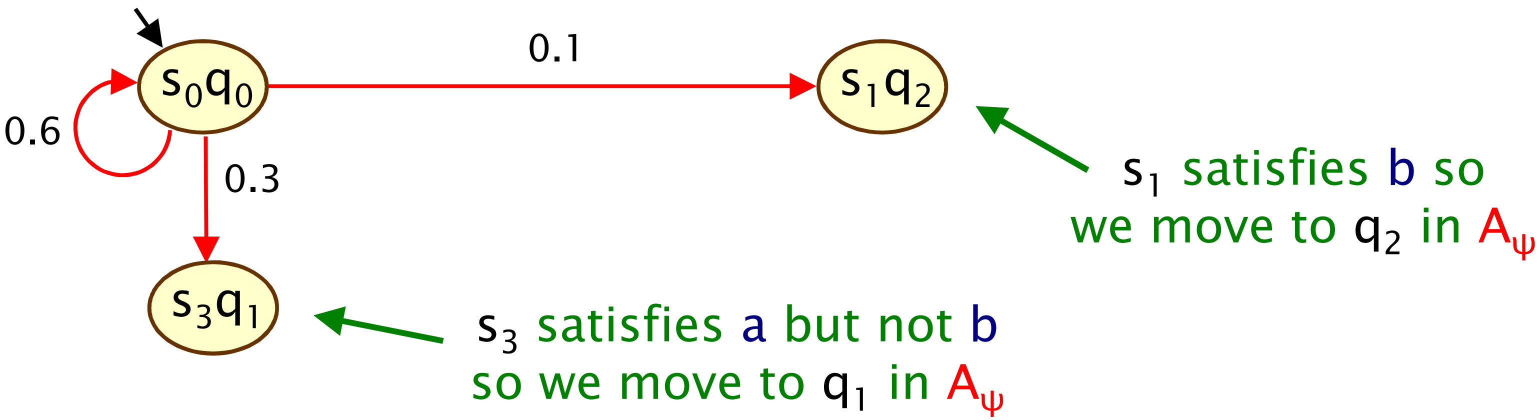
DTMC D



DRA A_ψ for $\psi = G\neg b \wedge GF\, a$

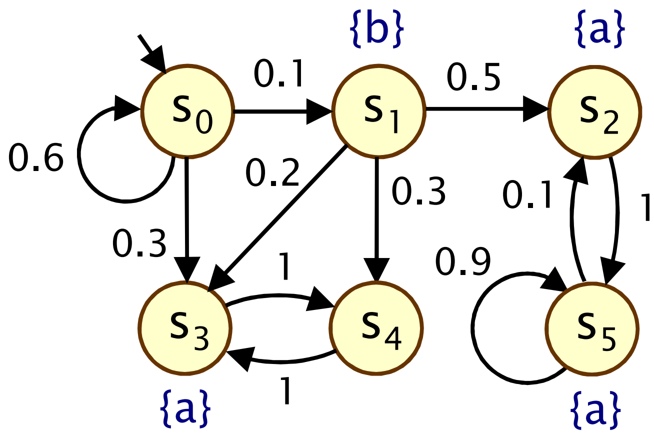


Product DTMC $D \otimes A_\psi$

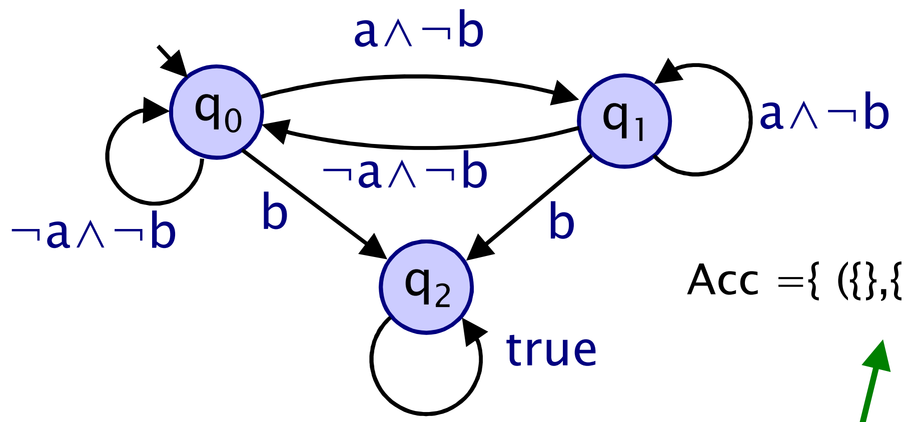


Example – Product DTMC

DTMC D



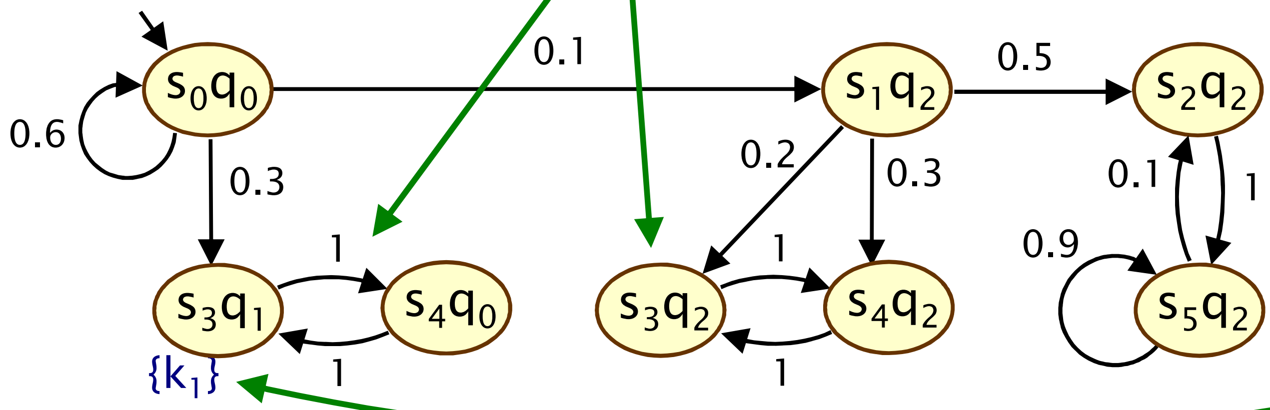
DRA A_ψ for $\psi = G\neg b \wedge GF\ a$



Acc ={ ({},{q_1}) }

Product DTMC $D \otimes A_\psi$

2 copies of s_3/s_4, one after seeing a b and one no b's



label states satisfying acceptance pair (L_1,K_1)

Product DTMC for a DRA

- For DTMC D and DRA A

$$\text{Prob}^{D}(s, A) = \text{Prob}^{D\otimes A}((s,q_s), \vee_{1\leq i\leq k} (FG \neg l_i \wedge GF\ k_i))$$

  - where $q_s = \delta(q_0, L(s))$
- Hence:

$$\text{Prob}^{D}(s, A) = \text{Prob}^{D\otimes A}((s,q_s), F\ T_{Acc})$$

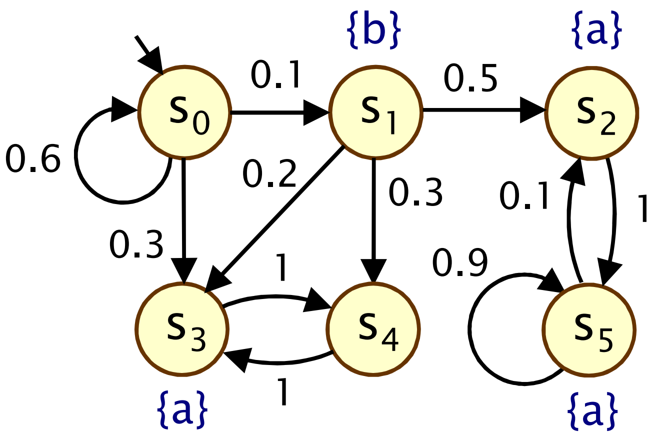
  - where T_{Acc} is the union of all accepting BSCCs in $D\otimes A$
  - an accepting BSCC T of $D\otimes A$ is such that, for some $1\leq i\leq k$, no states in T satisfy l_i and some state in T satisfies k_i

- Reduces to computing BSCCs and reachability probabilities

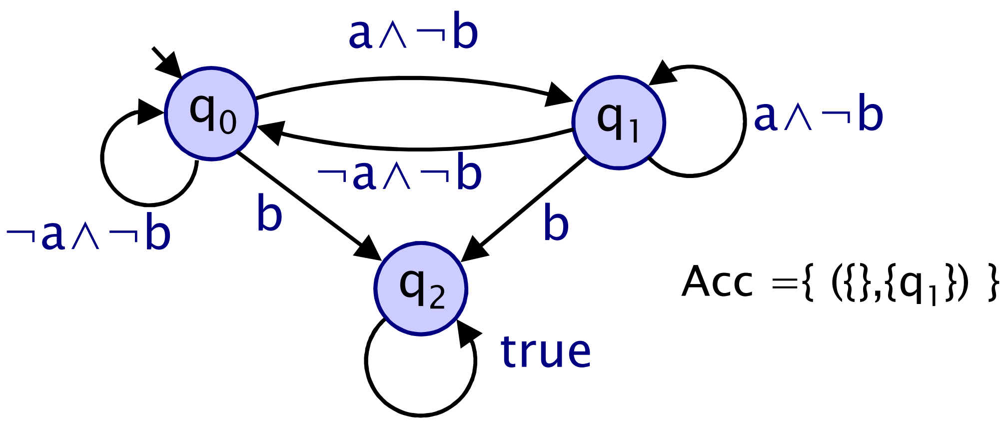
Example: LTL for DTMCs

- Compute Prob(s_0, G¬b ∧ GF a) for DTMC D:

DTMC D

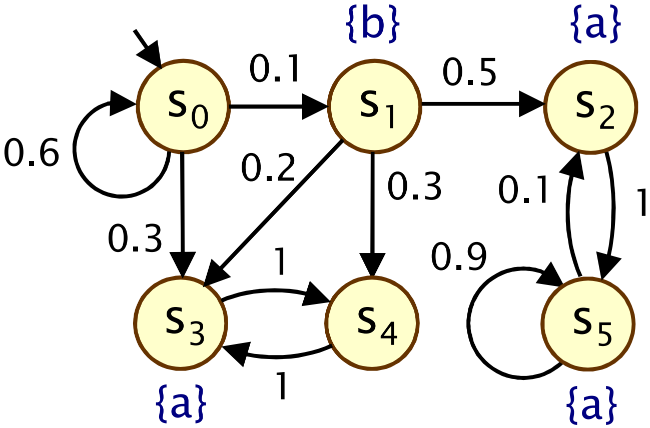


DRA A_ψ for ψ = G¬b ∧ GF a

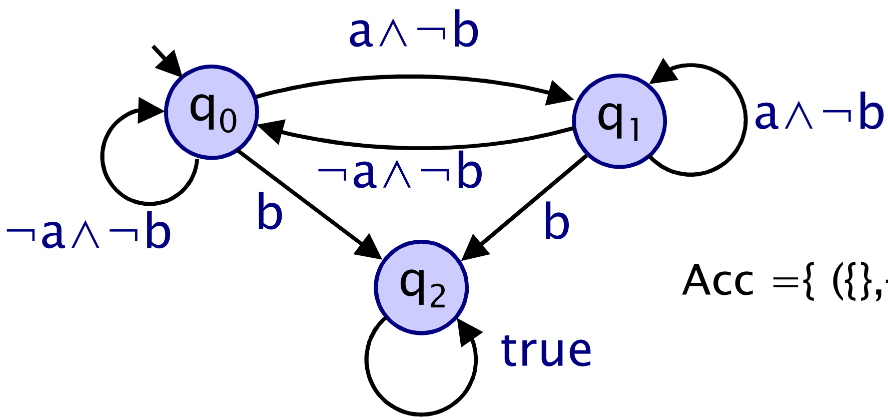


Example: LTL for DTMCs

DTMC D

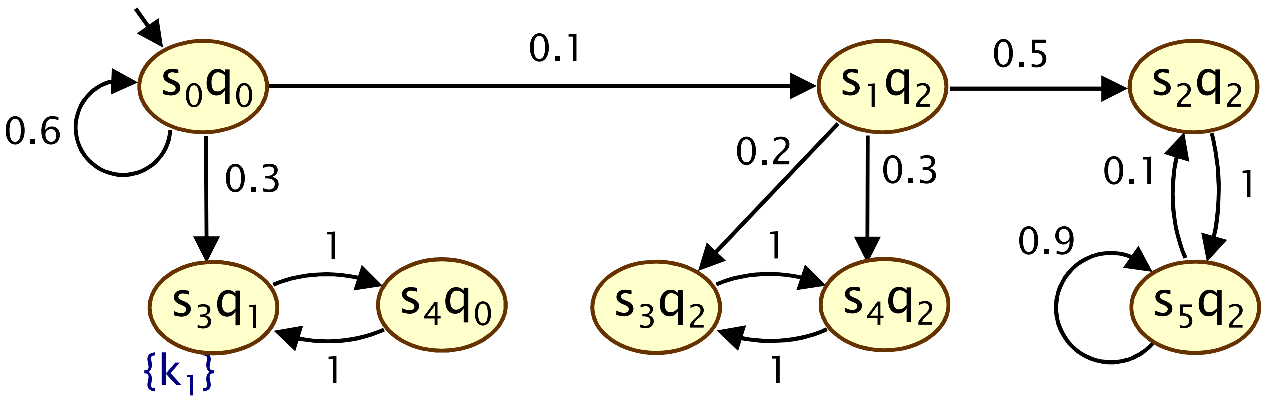


DRA A_ψ for $\psi = G\neg b \wedge GF\ a$



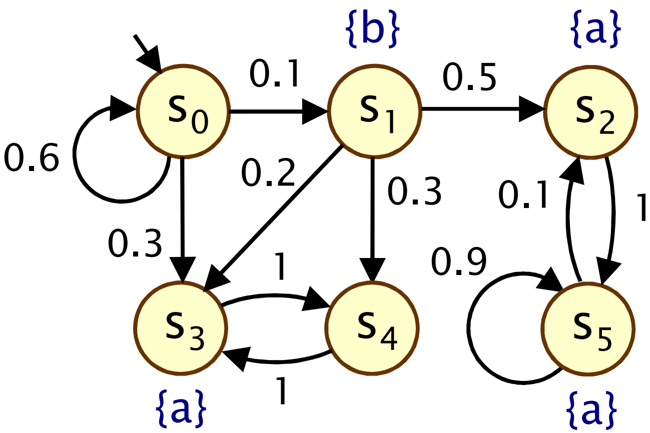
Acc = { ({},{q_1}) }

Product DTMC $D \otimes A_\psi$

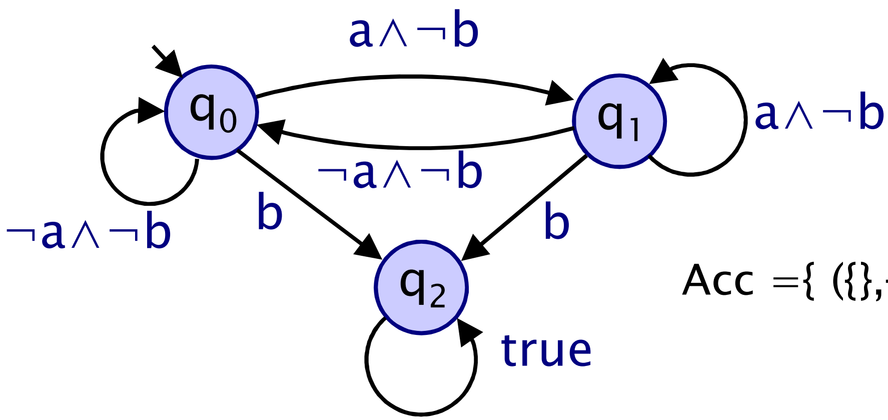


Example: LTL for DTMCs

DTMC D



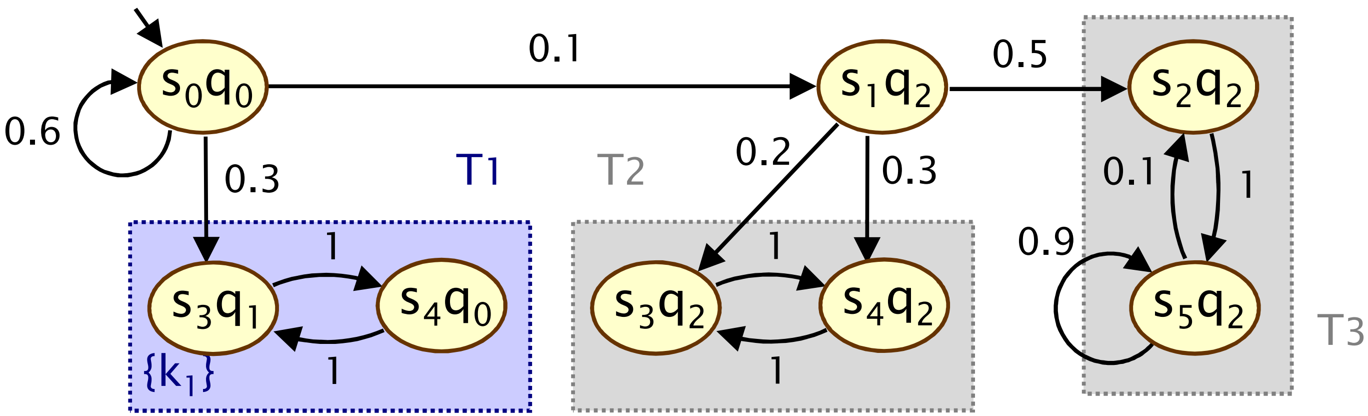
DRA A_ψ for ψ = G¬b ∧ GF a



Acc ={ ({},{q_1}) }

Product DTMC $D \otimes A_\psi$

$Prob^D(s_0, \psi) = Prob^{D \otimes A\psi}(s_0q_0, F\ T_1) = 3/4$



Complexity of LTL model checking

- Complexity of model checking LTL formula ψ on DTMC D
  - is doubly exponential in |ψ| and polynomial in |D|
  - (for the algorithm presented in these lectures)
- Double exponential blow-up comes from use of DRAs
  - size of NBA can be exponential in |ψ|
  - and DRA can be exponentially bigger than NBA
  - in practice, this does not occur and ψ is small anyway
- Polynomial-time operations required on product model
  - BSCC computation – linear in (product) model size
  - probabilistic reachability – cubic in (product) model size
- In total: $O(poly(|D|,|A_\psi|))$

- Complexity can be reduced to single exponential in |ψ|
  - see e.g. [CY88,CY95]

PCTL* model checking

- PCTL* syntax:
  - ϕ ::= true | a | $\phi \wedge \phi$ | $\neg\phi$ | $P_{\sim p}$ [ψ]
  - ψ ::= ϕ | $\psi \wedge \psi$ | $\neg\psi$ | X ψ | ψ U ψ
- Example:
  - $P_{>p}$ [GF (send $\rightarrow$ $P_{>0}$ [F ack])]

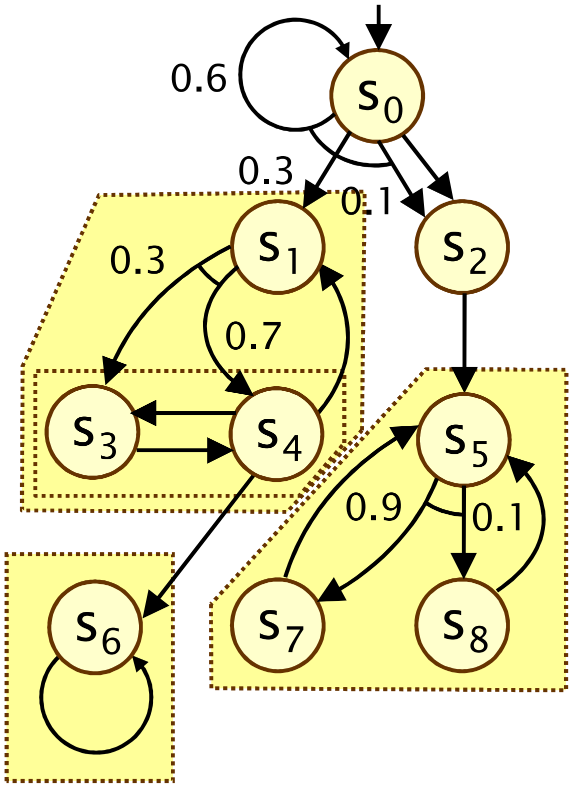
- PCTL* model checking algorithm
  - bottom-up traversal of parse tree for formula (like PCTL)
  - to model check $P_{\sim p}$ [ψ]:
    - replace maximal state subformulae with atomic propositions
    - (state subformulae already model checked recursively)
    - modified formula ψ is now an LTL formula
    - which can be model checked as for LTL

Overview (Part 3)

- Linear temporal logic (LTL)
- Strongly connected components
- ω-automata (Büchi, Rabin)
- LTL model checking for DTMCs
- LTL model checking for MDPs
- New developments and beyond PRISM

End components in MDPs

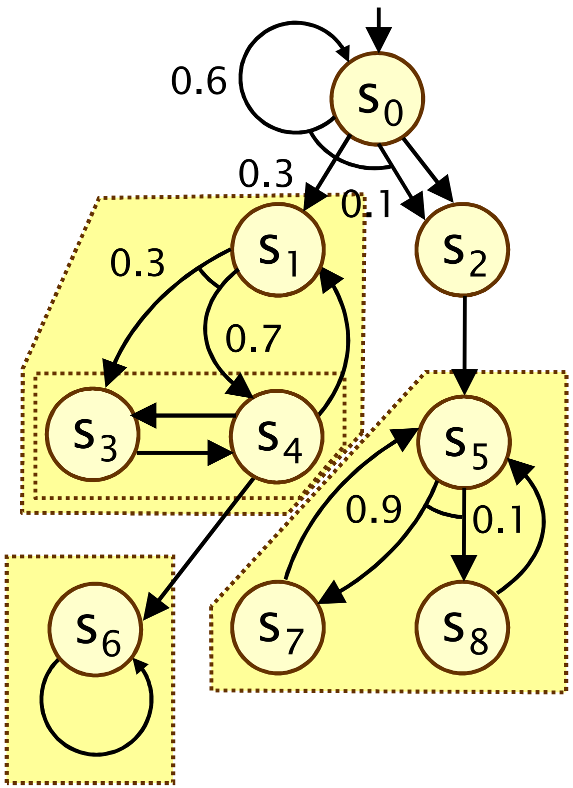
- End components of MDPs are the analogue of BSCCs in DTMCs
- An end component is a strongly connected sub-MDP
- A sub-MDP comprises a subset of states and a subset of the actions/distributions available in those states, which is closed under probabilistic branching



Note:
- action labels omitted
- probabilities omitted where =1

Recall – end components in MDPs

- End components of MDPs are the analogue of BSCCs in DTMCs
- For every end component, there is an adversary which, with probability 1, forces the MDP to remain in the end component, and visit all its states infinitely often
- Under every adversary σ, with probability 1 some end component will be reached and all of its states visited infinitely often (union of ECs reached with prob 1)



Long-run properties of MDPs

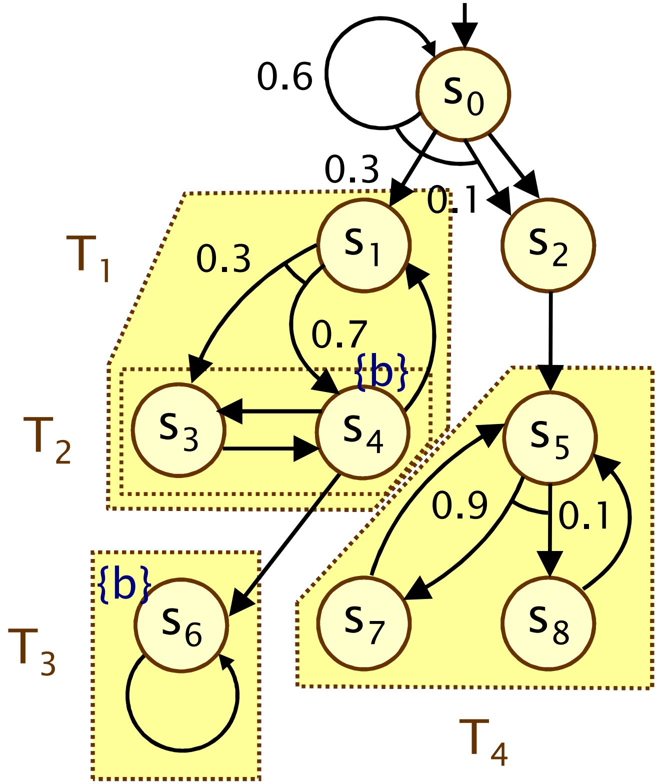
- Maximum probabilities
  - $p_{max}(s, GF\ a) = p_{max}(s, F\ T_{GFa})$
    - where T_{GFa} is the union of sets T for all end components (T,Steps’) with $T \cap Sat(a) \neq \varnothing$
  - $p_{max}(s, FG\ a) = p_{max}(s, F\ T_{FGa})$
    - where T_{FGa} is the union of sets T for all end components (T,Steps’) with $T \subseteq Sat(a)$
- Minimum probabilities
  - need to compute from maximum probabilities…
  - $p_{min}(s, GF\ a) = 1 - p_{max}(s, FG\neg a)$
  - $p_{min}(s, FG\ a) = 1 - p_{max}(s, GF\neg a)$

Example

- Model check: $P_{<0.8}$ [GF b] for s_0

- Compute p_{max}(GF b)
  - p_{max}(GF b) = p_{max}(s, F T_{GFb})
  - T_{GFb} is the union of sets T for all end components with T ∩ Sat(b) ≠ ∅
  - Sat(b) = { s_4, s_6 }
  - T_{GFb} = $T_1 \cup T_2 \cup T_3$ = { s_1, s_3 s_4, s_6 }
  - p_{max}(s, F T_{GFb}) = 0.75
  - p_{max}(GF b) = 0.75

- Result: $s_0 \vDash P_{<0.8}$ [GF b]



Automata-based properties for MDPs

- For an MDP M and automaton A over alphabet 2^{AP}
  - consider probability of "satisfying" language $L(A) \subseteq (2^{AP})^{\omega}$
  - $Prob^{M,adv}(s, P) = Pr_s^{M,adv} \{ \omega \in Path^{M,adv}(s) \mid trace(\omega) \in L(A) \}$
  - $p_{max}^{M}(s, A) = \sup_{adv \in Adv} Prob^{M,adv}(s, A)$
  - $p_{min}^{M}(s, A) = \inf_{adv \in Adv} Prob^{M,adv}(s, A)$

- Might need minimum or maximum probabilities
  - e.g. $s \models P_{\geq 0.99} [\ \psi_{good}\ ] \Leftrightarrow p_{min}^{M}(s, \psi_{good}) \geq 0.99$
  - e.g. $s \models P_{\leq 0.05} [\ \psi_{bad}\ ] \Leftrightarrow p_{max}^{M}(s, \psi_{bad}) \leq 0.05$
- But, ψ-regular properties are closed under negation
  - as are the automata that represent them
  - so can always consider maximum probabilities…
  - $p_{max}^{M}(s, \psi_{bad})$ or $1 - p_{max}^{M}(s, \neg\psi_{good})$

LTL model checking for MDPs

- Model check LTL specification $P_{\sim p}$ [ψ] against MDP M
- 1. Convert problem to one needing maximum probabilities
  - e.g. convert $P_{>p}$ [ψ] to $P_{<1-p}$ [¬ψ]
- 2. Generate a DRA for ψ (or ¬ψ)
  - build nondeterministic Büchi automaton (NBA) for ψ [VW94]
  - convert the NBA to a DRA [Saf88]
- 3. Construct product MDP M⊗A
- 4. Identify accepting end components (ECs) of M⊗A
- 5. Compute max. probability of reaching accepting ECs
  - from all states of the D⊗A
- 6. Compare probability for (s, q_s) against p for each s

Product MDP for a DRA

- For an MDP $M = (S, s_{init}, Steps, L)$
- and a (total) DRA $A = (Q, \Sigma, \delta, q_0, Acc)$
  - where $Acc = \{ (L_i, K_i) \mid 1 \leq i \leq k \}$

- The product MDP $M \otimes A$ is:
  - the MDP $(S \times Q, (s_{init}, q_{init}), Steps', L')$ where:

$$q_{init} = \delta(q_0, L(s_{init}))$$

$$Steps'(s,q) = \{ \mu^q \mid \mu \in Step(s) \}$$

$$\mu^q(s', q') = \begin{cases} \mu(s') & \text{if } q' = \delta(q, L(s)) \\ 0 & \text{otherwise} \end{cases}$$

$l_i \in L'(s,q)$ if $q \in L_i$ and $k_i \in L'(s,q)$ if $q \in K_i$

(i.e. state sets of acceptance condition used as labels)

Product MDP for a DRA

- For MDP M and DRA A

$$p_{max}^{M}(s, A) = p_{max}^{M \otimes A}((s,q_s), \vee_{1 \leq i \leq k} (FG \neg l_i \wedge GF k_i))$$

  - where $q_s = \delta(q_0, L(s))$

- Hence:

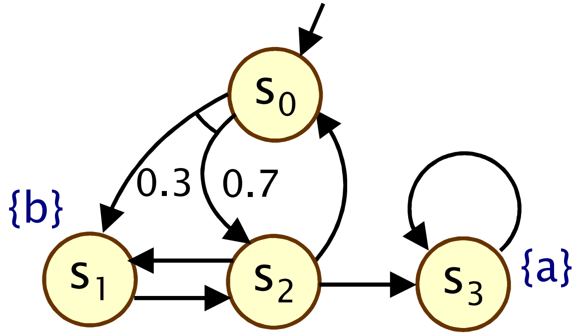
$$p_{max}^{M}(s, A) = p_{max}^{M \otimes A}((s,q_s), F\ T_{Acc})$$

  - where T_{Acc} is the union of all sets T for accepting end components (T,Steps') in D⊗A
  - an accepting end components is such that, for some $1 \leq i \leq k$:
    - $q \vDash \neg l_i$ for all $(s,q) \in T$ and $q \vDash k_i$ for some $(s,q) \in T$
    - i.e. $T \cap (S \times L_i) = \emptyset$ and $T \cap (S \times K_i) \neq \emptyset$

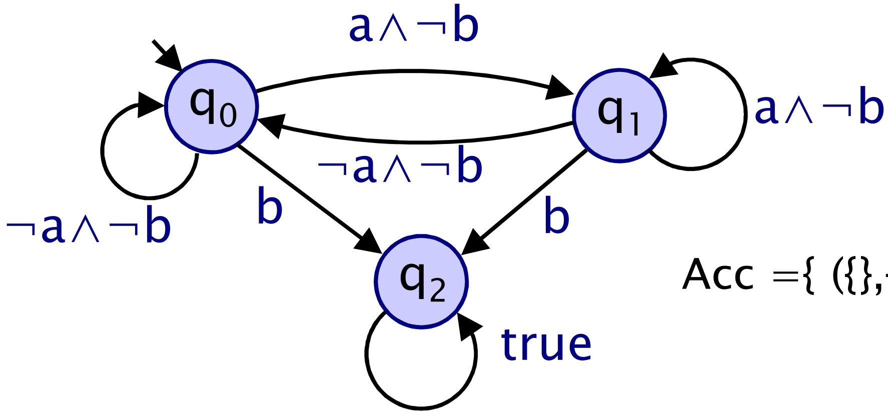
Example: LTL for MDPs

- Model check $P_{<0.8}$ [G ¬b ∧ GF a] for MDP M:
  - need to compute $\underline{p}_{max}(s_0, G\ \neg b \wedge GF\ a)$

MDP M



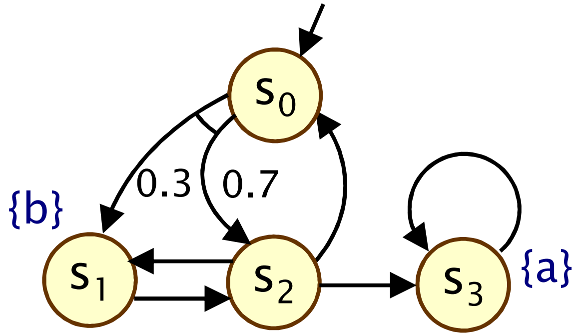
DRA A_ψ for $\psi = G\neg b \wedge GF\ a$



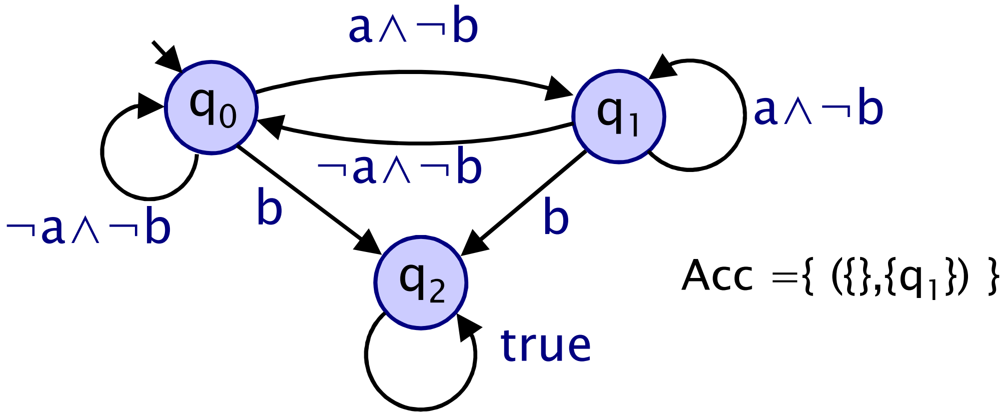
Acc ={ ({},{q_1}) }

Example: LTL for MDPs

MDP M

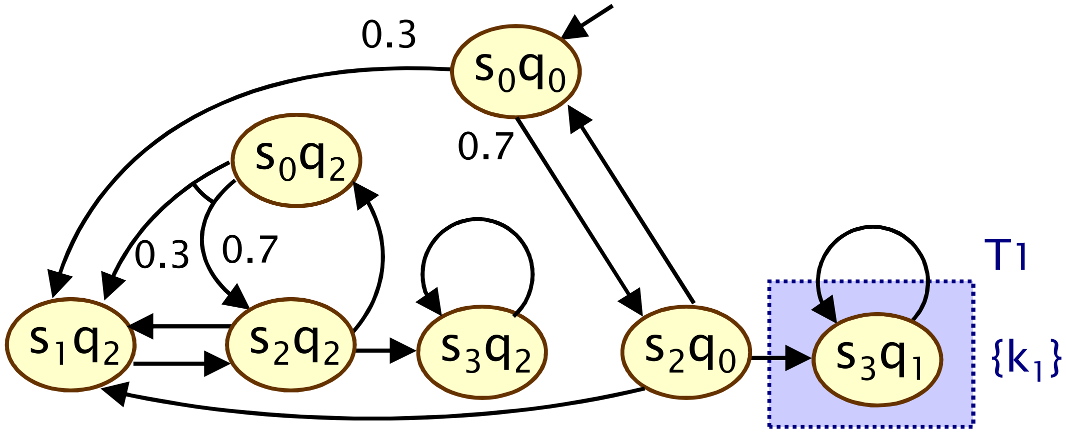


DRA A_ψ for $\psi = G\neg b \wedge GF\, a$



Product MDP $M \otimes A_\psi$

$p_{max}{}^{M}(s_0, \psi) = p_{max}{}^{M\otimes A\psi}(s_0q_0, F\ T_1) = 0.7$



LTL model checking for MDPs

- Complexity of model checking LTL formula ψ on MDP M
  - is doubly exponential in |ψ| and polynomial in |M|
  - unlike DTMCs, this cannot be improved upon

- PCTL* model checking
  - LTL model checking can be adapted to PCTL*, as for DTMCs

- Maximal end components
  - can optimise LTL model checking using maximal end components (there may be exponentially many ECs)

- Optimal adversaries for LTL formulae
  - e.g. memoryless adversary always exists for $p_{max}(s, GF\ a)$, but not for $p_{max}(s, FG\ a)$

Summary (LTL model checking)

- Linear temporal logic (LTL)
  - combines path operators; PCTL* subsumes LTL and PCTL
- ω-automata: represent ω-regular languages/properties
  - can translate any LTL formula into a Büchi automaton
  - for deterministic ω-automata, we use Rabin automata
- Long-run properties of DTMCs
  - need bottom strongly connected components (BSCCs)
- LTL model checking for DTMCs
  - construct product of DTMC and Rabin automaton
  - identify accepting BSCCs, compute reachability probability
- LTL model checking for MDPs
  - MDP-DRA product, reachability of accepting end components

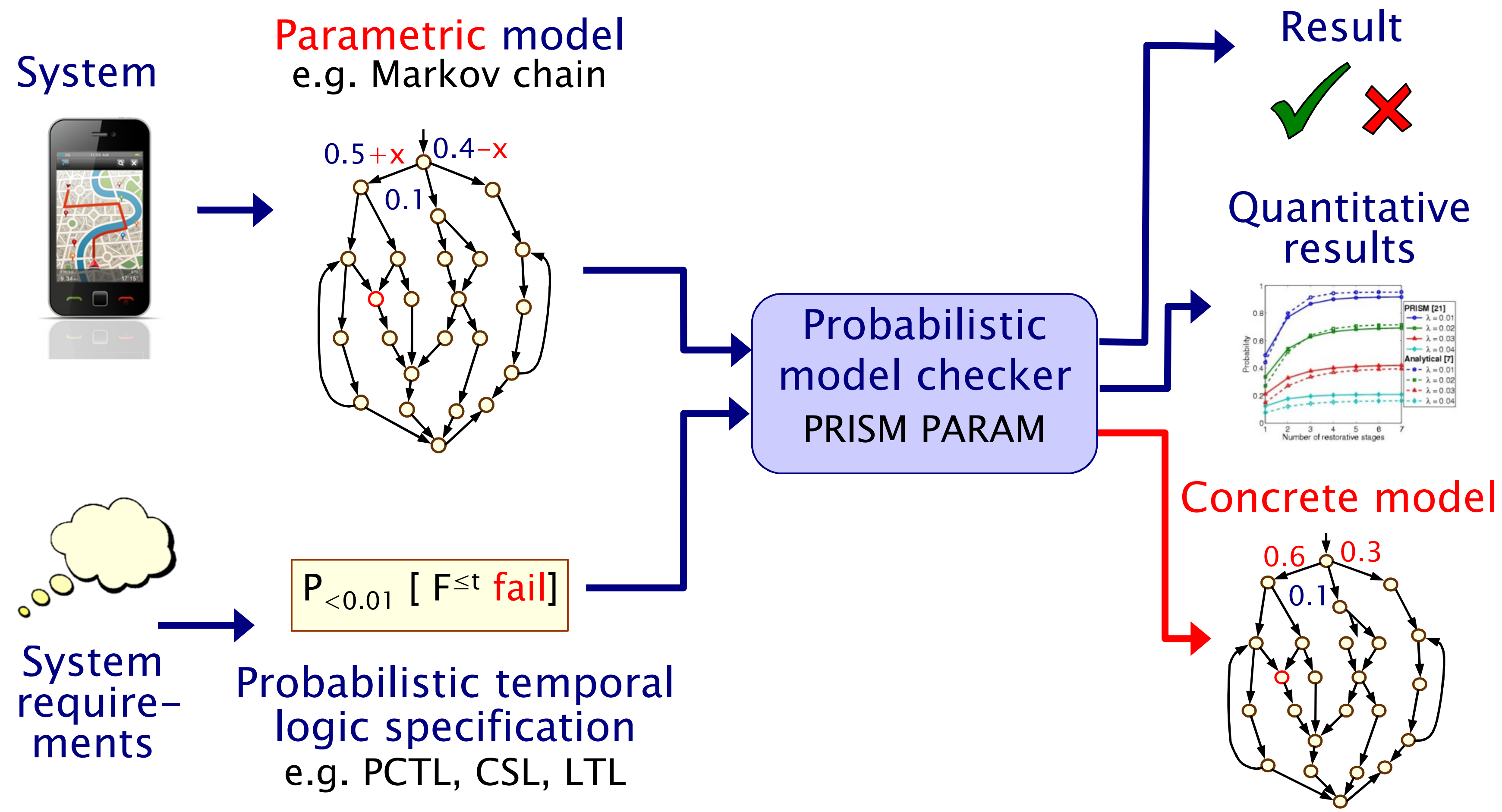
PRISM: Recent & new developments

- New features:
  1. parametric model checking
  2. parameter synthesis
  3. strategy synthesis
  4. stochastic multi-player games
  5. real-time: probabilistic timed automata (PTAs)

- Further new additions:
  – enhanced statistical model checking
    (approximations + confidence intervals, acceptance sampling)
  – efficient CTMC model checking (fast adaptive uniformisation)
  – benchmark suite & testing functionality
  – www.prismmodelchecker.org

– Beyond PRISM…

Parametric model checking and synthesis



1. Parametric model checking in PRISM

- Parametric Markov chain models in PRISM
  - probabilistic parameters expressed as unevaluated constants
  - e.g. const double x;
  - transition probabilities are expressions over parameters, e.g. 0.4 + x
- Properties are given in PCTL, with parameter constants
  - new construct constfilter (min, x1*x2, phi)
  - filters over parameter values, rather than states
- Implemented in 'explicit' engine
  - returns mapping from parameter regions (e.g. [0.2,0.3],[-2,0]) to rational functions over the parameters
  - filter properties used to find parameter values that optimise the function
  - reimplementation of PARAM 2.0 [Hahn et al]

2. Parameter synthesis

- Find optimal parameter value given a parametric model and PCTL/CSL property
  - parametric probabilities and rates
- Techniques
  - discretisation and integer parameters
  - constraint solving, including parametric symbolic constraints
  - iterative refinement to improve accuracy
  - sampling to improve efficiency
  - but scalability is still the biggest challenge
- Implementation
  - using tool combination involving Z3, python, PRISM
  - see also Prophecy from Katoen's group

3. Controller (strategy) synthesis

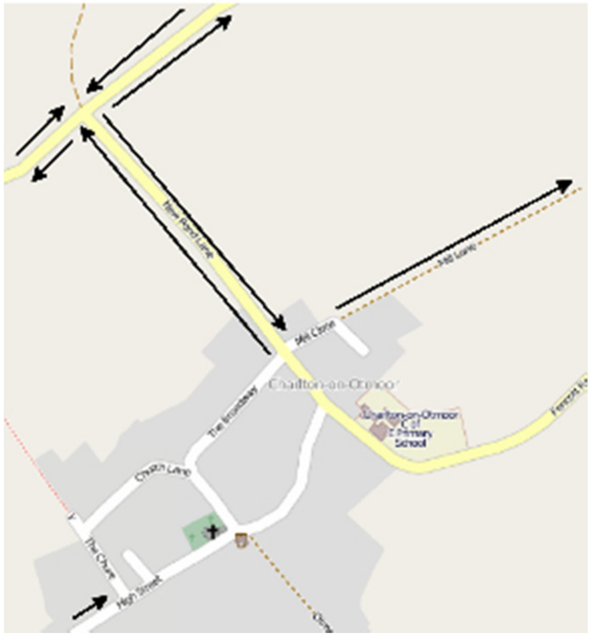
- Can synthesise permissive controllers [TACAS14]
  - a permissive controller allows more than one action per state
  - adds flexibility in case an action become temporarily unavailable, improving robustness
  - e.g. StockPrice Viewer (Android)
  - expressed in terms of multi-strategies
- Can synthesise controllers using machine learning [ATVA14]
  - partial exploration of the state space, with guarantees of accuracy
  - combines real-time dynamic programming with value iteration
  - focus on updating "most important parts" = most often visited by good strategies
  - speeds up value iteration
- Implemented in PRISM for both MDPs and SMGs

4. Stochastic multi-player games

- Extension of PRISM
  - modelling of stochastic multi-player games
  - probabilistic model checking of rPATL and extensions
  - strategy synthesis and analysis
    - optimal strategy generation
    - strategy simulation and export
    - model checking of applied strategies
  - graphical user interface (editors, simulator, graph plotting, …)
- PRISM-games 2.0:
  - long-run average and ratio properties
  - multi-objective strategy synthesis
  - Pareto curve generation and visualisation
  - compositional strategy synthesis techniques
- Available from http://www.prismmodelchecker.org/games/

Case study: Autonomous urban driving

- Inspired by DARPA challenge
  - represent map data as a stochastic game, with environment active, able to select hazards
  - express goals as conjunctions of probabilistic and reward properties
  - e.g. “maximise probability of avoiding hazards and minimise time to reach destination”



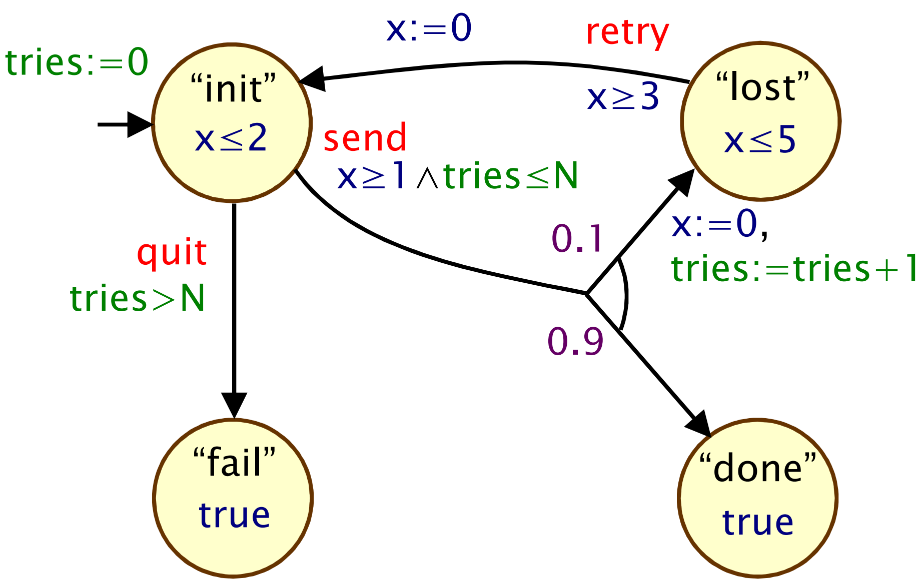
- Solution (PRISM-games 2.0)
  - synthesise a probabilistic strategy to achieve the multi-objective goal
  - enable the exploration of trade-offs between subgoals
  - applied to synthesise driving strategies for English villages

Synthesis for Multi-Objective Stochastic Games: An Application to Autonomous Urban Driving, Chen et al., In *Proc* QEST 2013

5. Probabilistic timed automata (PTAs)

- Probability + nondeterminism + real-time
  - timed automata + discrete probabilistic choice, or…
  - probabilistic automata + real-valued clocks

- PTA example: message transmission over faulty channel



States
- locations + data variables

Transitions
- guards and action labels

Real-valued clocks
- state invariants, guards, resets

Probability
- discrete probabilistic choice

Modelling PTAs in PRISM

- PRISM modelling language
  - textual language, based on guarded commands

```
pta
const int N;
module transmitter
    s : [0..3] init 0;
    tries : [0..N+1] init 0;
    x : clock;
    invariant (s=0 ⇒ x≤2) & (s=1 ⇒ x≤5) endinvariant
    [send] s=0 & tries≤N & x≥1
           → 0.9 : (s'=3)
           + 0.1 : (s'=1) & (tries'=tries+1) & (x'=0);
    [retry] s=1 & x≥3 → (s' =0) & (x' =0);
    [quit]  s=0 & tries>N → (s' =2);
endmodule
rewards "energy" (s=0) : 2.5; endrewards
```

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Basic ingredients:

- modules
- variables
- commands

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Basic ingredients:

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New for PTAs:

- clocks
- invariants
- guards/resets

Modelling PTAs in PRISM

- PRISM modelling language
  - textual language, based on guarded commands

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```

Basic ingredients:

- modules
- variables
- commands

New for PTAs:

- clocks
- invariants
- guards/resets

Also:

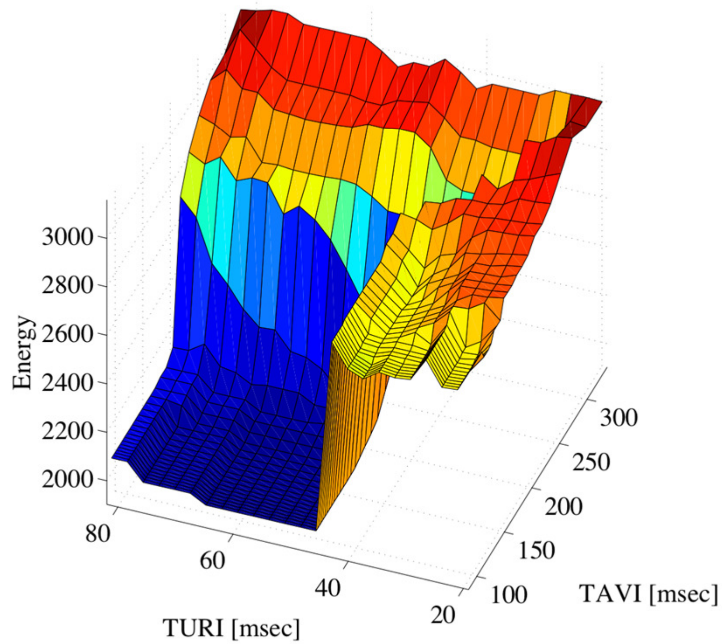
- rewards (i.e. costs, prices)
- parallel composition

Model checking PTAs in PRISM

- Properties for PTAs:
  - min/max probability of reaching X (within time T)
  - min/max expected cost/reward to reach X
    (for “linearly-priced” PTAs, i.e. reward gain linear with time)
- PRISM has two different PTA model checking techniques…
- “Digital clocks” – conversion to finite-state MDP
  - preserves min/max probability + expected cost/reward/price
  - (for PTAs with closed, diagonal-free constraints)
  - efficient, in combination with PRISM’s symbolic engines
- Quantitative abstraction refinement
  - zone-based abstractions of PTAs using stochastic games
  - provide lower/upper bounds on quantitative properties
  - automatic iterative abstraction refinement

Beyond PRISM: Cardiac pacemaker

- Develop model-based framework
  - timed automata model for pacemaker software [Jiang et al]
  - hybrid heart models in Simulink, adopt synthetic ECG model (non-linear ODE) [Clifford et al]
- Properties
  - (basic safety) maintain 60-100 beats per minute
  - (advanced) detailed analysis energy usage, plotted against timing parameters of the pacemaker
  - parameter synthesis: find values for timing delays that optimise energy usage



Optimal timing delays problem

- Optimal timing delay synthesis for timed automata [EMSOFT2014][HSB 2015]
- The parameter synthesis problem solved is
  - given a parametric network of timed I/O automata, set of controllable and uncontrollable parameters, CMTL property ϕ and length of path n
  - find the optimal controllable parameter values, for any uncontrollable parameter values, with respect to an objective function O, such that the property ϕ is satisfied on paths of length n, if such values exist
- Consider family of objective functions
  - maximise volume, minimise energy
- Discretise parameters, assume bounded integer parameter space and path length
  - decidable but high complexity (high time constants)

Optimal probability of timing delays

- Previously, no nondeterminism and no probability in the model considered
- Consider parametric probabilistic timed automata (PPTA),
  - e.g. guards of the form $x \leq b$,
- Can we synthesise optimal timing parameters to optimise the reachability probability?
- Semi-algorithm [RP 2014]
  - exploration of parametric symbolic states, i.e. location, time zone and parameter valuations
  - forward exploration only gives upper bounds on maximum probability (resp. lower for minimum)
  - but stochastic game abstraction yields the precise solution…
- Implementation in progress

Quantitative verification – Trends

- Being 'younger', generally lags behind conventional verification
  - much smaller model capacity
  - compositional reasoning in infancy
  - automation of model extraction/adaptation very limited

- Tool usage on the increase, in academic/industrial contexts
  - real-time verification/synthesis in embedded systems
  - probabilistic verification in security, reliability, performance

- Shift towards greater automation
  - specification mining, model extraction, synthesis, verification, …

- But many challenges remain!

Acknowledgements

- My group and collaborators in this work
- Project funding
  - ERC, EPSRC, Microsoft Research
  - Oxford Martin School, Institute for the Future of Computing

- See also
  - VERIWARE www.veriware.org
  - PRISM www.prismmodelchecker.org

PhD Comics and Oxford…

- You are welcome to visit Oxford!
- PhD scholarships, postdocs in verification and synthesis, and more

Thank you for your attention

More info here:
www.prismmodelchecker.org