/* BYZANTINE AGREEMENT PROTOCOL */
/* this file contains the template of the byzantine agreement protocol */
/* gen 7/12/2001 */
/* replaced the probabilistic choice of the coins by a non-deterministic choice */
/* and suppose that the coin in round \( r \) is flipped when at least \( n - 2t \) honest parties have read the main votes in round \( r \) */
/* note that the coin can never be tossed earlier than this since \( n - t \) shares are required before it can be revealed */
/* note that we have change the notations for the coins: for round \( r \) the coin is denoted \( \text{coin}[r+1] \) */
/* this is to simplify the code: it allows the pre-votes for each round to be based on the coin for the previous round */
/* by setting the coin for round -1 (coin[0]) always to 0 */

/* instead of parties reading votes individually, we store the total number of honest votes for each preference */
/* and for corrupted parties we include only boolean variables indicating what corrupted parties can vote for */
/* recalling that different parties may see corrupted parties vote for different values */
/* based on these totals we then know what combination of votes each party can read */
/* that is what the adversary can make parties vote for */
/* note that sometimes there will be a non-deterministic choice as to what a party will vote for */
/* this is a choice of the adversary since it depends on which votes a party reads */

/* an honest party must get at least \( n - 2t \) votes from other honest parties */
/* and the remaining votes can come from either honest or corrupted parties */
/* if we can show: “if an honest party has a vote for a value then corrupted parties can also vote for this value” */
/* we need only count up to \( n - 2t \) honest party votes and use corrupted parties for the remaining votes */
/* this has been proved (properties corrupted5 and corrupted6 in lemmas.smv) */

/* note that to deal with the validity part of the proof and to keep things consistent the initial round is 0 instead of 1 */
/* this is because the round variable is an ordset and cannot specify round=1, but we can specify round=0 */

/* one further change is that we let the pre-votes be cast before they are cast in the actual protocol */
/* now they are cast when a party reads the main-votes of the previous round */
/* note that this can only give the adversary more power */
/* however, in the original protocol at the time when the pre-votes are now cast */
/* an honest party has read the main-votes it will use to cast a pre-vote */
/* therefore at this time the adversary already knows what pre-vote the party is going make */
/* and hence moving the casting of pre-votes to this point makes no difference to what the adversary knows */
/* there is one added complication: we need to store two different possible pre-votes which depend on */
/* what value of the corresponding coin becomes when it is tossed */

/* CONSTANTS \((n=10 \& \ t=3)\) */
#define N 7 /* number of honest parties */
#define M 4 /* \( M = n - 2t \) number of honest pre and main votes that must be read */
#define K 4 /* \( K = 2t + 1 - t = t + 1 \) number of honest pre processing votes that must be read */

/* ABSTRACT DATA TYPES */
ordset PROC 1..N; /* set of parties */
ordset VOTES 0..M; /* honest pre and main votes */
ordset PRE_PROC_VOTES 0..K; /* honest preprocessing votes */
ordset ROUNDS 0..; /* round numbers */

data PC = {INITIAL, PRE_PROCESSING, MAIN, DECIDE, COIN, DONE};
/* INITIAL - make preprocessing vote (initial state) */
/* PRE_PROCESSING - read preprocessing votes and make initial pre-vote */
/* MAIN - read pre-votes and make main vote */
/* DECIDE - read main votes and make choice whether to decide or not and next pre-vote */
/* COIN - generate share of the coin */
/* DONE - finished */
module main(act){

/* —————————INPUTS———————————————*/

act : PROC; /* SCHEDULER left unspecified */

/* in certain cases a party can cast different votes depending on which votes it reads */
/* we represent these choices using the following variables */
/* note since this is decided by the adversary we leave these variables unspecified */
/* pre votes */
nd_pre : {0,1}; /* non deterministic choice between voting for 0 and 1 */
nd_pre1 : {0,2}; /* non deterministic choice between voting for 0 and coin */
nd_pre2 : {1,2}; /* non deterministic choice between voting for 1 and coin */
nd_pre3 : {0,1,2}; /* non deterministic choice between voting for 1 and coin */
/* main votes */
nd_main1 : {0,2}; /* non deterministic choice between voting for 0 and 2 */
nd_main2 : {1,2}; /* non deterministic choice between voting for 1 and 2 */

start : array PROC of 0..1; /* initial values of parties */
forall (i in PROC) next(start[i]) := start[i]; /* initial values do not change (needed for validity) */

/* ——————THE PROTOCOL FOR HONEST PARTIES———————*/

/* PHASE */
pc : array PROC of PC; /* pc[i] - phase of party i */

/* VALUES OF VOTES */
/* pre-processing-votes: 0 or 1 */
/* pre-votes: 0 or 1 */
/* main-votes: 0, 1 or 2 where 2 denotes abstain */

/* PRE_PROCESSING */
preproc : PRE_PROC_VOTES; /* preproc - number of pre processing votes */
preproc_votes : array 0..1 of PRE_PROC_VOTES; /* preproc_votes[v] - number of pre processing votes for v */

/* PRE_VOTE */
pre : array ROUNDS of VOTES; /* pre[r] - number of pre-votes in round r */
pre_votes : array ROUNDS of array 0..1 of array 0..1 of VOTES;
/* pre_votes[r][c][v] - number of pre-votes for v in round r if the coin in round r equals c */

/* MAIN_VOTE */
main : array ROUNDS of VOTES; /* main[r] - number of main-votes so far in round r */
main_votes : array ROUNDS of array 0..2 of VOTES; /* main_votes[r][v] - number of main-votes so far for v in round r */

/* ROUND */
round : array PROC of ROUNDS; /* round[i] - round party i is currently in */

/* DECIDING */
declare : array ROUNDS of array PROC of boolean; /* declare[r][i] - party i decided in round r */
declare_value : array ROUNDS of array PROC of 0..1;
/* declare[r][i][v]=v what value party i decided on in round r (if the party decides) */

/* COIN */
coin : array ROUNDS of {not_tossed, 0, 1}; /* coin[r] value of the coin in round r */

/* INITIAL VALUES */
forall (i in PROC) {

}
init(pc[i]) := INITIAL;
init(round[i]) := 0;
}

/* set all totals to be zero */
init(pre_proc) := 0;
for(v = 0; v ≤ 1; v = v + 1) init(pre_proc_votes[v]) := 0;
forall(r in ROUNDS) {
  init(pre[r]) := 0;
  init(main[r]) := 0;
  for(c = 0; c ≤ 1; c = c + 1) for(v = 0; v ≤ 1; v = v + 1) init(pre_votes[r][c][v]) := 0;
  for(v = 0; v ≤ 2; v = v + 1) init(main_votes[r][v]) := 0;
}

/* initially undecided */
forall(r in ROUNDS) forall(i in PROC) init(decide[r][i]) := 0;

/* as explained the coin in round 0 is set to 0 then never changed */
/* the remaining coins initially are not tossed */
forall(r in ROUNDS) init(coin[r]) := (r=0) ? 0 : not_tossed;

/* NEXT VALUES */
switch (pc[act]) {
  INITIAL: {
    next(pc[act]) := PRE_PROCESSING; /* move to pre processing */
    /* make pre processing vote based on the value of start[act] */
    next(pre_proc_votes[start[act]]) := (pre_proc_votes[start[act]])=K ? pre_proc_votes[start[act]] : pre_proc_votes[start[act]]+1;
    /* record that a pre processing vote has been cast */
    next(pre_proc) := pre_proc=K ? pre_proc : pre_proc+1;
  }
  PRE_PROCESSING: {
    if (pre_proc=K) { /* sufficient pre processing votes */
      next(pc[act]) := MAIN; /* move to MAIN */
      /* record that a pre vote has been cast */
      next(pre[round[act]]) := pre[round[act]]=M ? pre[round[act]] : pre[round[act]]+1;
      /* sufficient votes to have a pre vote for either value therefore the vote is the adversary's choice */
      if (pre_proc_votes[0]>0 ∧ pre_proc_votes[1]>0) {
        next(pre_votes[round[act]][0][nd_pre]) :=
          pre_votes[round[act]][0][nd_pre]=M ? pre_votes[round[act]][0][nd_pre] : pre_votes[round[act]][0][nd_pre]+1;
        next(pre_votes[round[act]][1][nd_pre]) :=
          pre_votes[round[act]][1][nd_pre]=M ? pre_votes[round[act]][1][nd_pre] : pre_votes[round[act]][1][nd_pre]+1;
      }
      /* pre vote for 0 only */
      else if (pre_proc_votes[0]>0) {
        for(c = 0; c ≤ 1; c = c + 1)
          next(pre_votes[round[act]][c][0]) := pre_votes[round[act]][c][0]=M ? pre_votes[round[act]][c][0] : pre_votes[round[act]][c][0]+1;
      }
      /* pre vote for 1 only */
      else if (pre_proc_votes[1]>0) {
        for(c = 0; c ≤ 1; c = c + 1)
          next(pre_votes[round[act]][c][1]) := pre_votes[round[act]][c][1]=M ? pre_votes[round[act]][c][1] : pre_votes[round[act]][c][1]+1;
      }
      /* the above should be the only cases but as a default suppose the adversary can choose */
      else {
        for(c = 0; c ≤ 1; c = c + 1)
          next(pre_votes[round[act]][c][nd_pre]) :=
            pre_votes[round[act]][c][nd_pre]=M ? pre_votes[round[act]][c][nd_pre] : pre_votes[round[act]][c][nd_pre]+1;
      }
    }
  }
  MAIN: {
    /* read pre votes and cast main vote */
/* to simplify the conditions have used the assumptions that there cannot be M votes for two distinct values in the same round */
if (pre[round[act]] == M ∧ ¬coin[round[act]] == not_tossed) { /* sufficient pre votes and the coin has been tossed */
    next(pc[act]) := DECIDE; /* move to DECIDE */
}
/* record that the party has made a main vote */
next(main[round[act]]) := main[round[act]] = M ∧ main[round[act]] = main[round[act]] + 1;
/* main vote for 0 or abstain only */
if (pre_votes[round[act]][coin[round[act]]][0] = M ∧ corrupted_pre[round[act]][coin[round[act]]][0] ∧
    (pre_votes[round[act]][coin[round[act]]][1] > 0 ∨ corrupted_pre[round[act]][coin[round[act]]][1] ))
    next(main_votes[round[act]][nd_main1]) :=
    main_votes[round[act]][nd_main1] = M ∧ main_votes[round[act]][nd_main1] = main_votes[round[act]][nd_main1] + 1;
/* main vote for 0 or abstain only */
else if (pre_votes[round[act]][coin[round[act]]][1] = M ∧ corrupted_pre[round[act]][coin[round[act]]][1] ∧
    (pre_votes[round[act]][coin[round[act]]][0] > 0 ∨ corrupted_pre[round[act]][coin[round[act]]][0] ))
    next(main_votes[round[act]][nd_main2]) :=
/* main vote for 0 only */
else if (pre_votes[round[act]][coin[round[act]]][0] = M ∧ corrupted_pre[round[act]][coin[round[act]]][0])
    next(main_votes[round[act]][0]) := main_votes[round[act]][0] = M ∧ main_votes[round[act]][0] = main_votes[round[act]][0] + 1;
/* main vote for 1 only */
else if (pre_votes[round[act]][coin[round[act]]][1] = M ∧ corrupted_pre[round[act]][coin[round[act]]][1])
/* left with only a main vote for abstain */
else
}

DECIDE : {
/* to simplify the model we allow decide_value to be set without decide being set to true */
/* in the case the value of decide_value does not matter */
/* and again use the assumptions that there cannot be M votes for two distinct values in the same round */
if (main[round[act]] == M) { /* sufficient main votes */
    next(pc[act]) := COIN;
    /* record that a pre vote has been cast */
    next(pre[round[act]] + 1) := pre[round[act]] + 1 = M ∧ pre[round[act]] + 1 = pre[round[act]] + 1 + 1;
    /* must decide on 0 and cast a pre-vote for 0 */
    if (main_votes[round[act]][0] = M ∧ corrupted_main[round[act]][0] ∧ main_votes[round[act]][1] = 0 ∧
        ¬corrupted_main[round[act]][1] ∧ main_votes[round[act]][2] = 0 ∧ ¬corrupted_main[round[act]][2] )
        next(decide[round[act]][act]) := 1;
    next(decide_value[round[act]][act]) := 0;
    for (c = 0; c ≤ 1; c = c + 1)
        next(pre_votes[round[act]] + 1)[c][0] := pre_votes[round[act]] + 1)[c][0] = pre_votes[round[act]] + 1)[c][0] + 1;
        /* must decide on 1 and cast a pre-vote for 1 */
    else if (main_votes[round[act]][1] = M ∧ corrupted_main[round[act]][1] ∧ main_votes[round[act]][0] = 0 ∧
        ¬corrupted_main[round[act]][0] ∧ main_votes[round[act]][2] = 0 ∧ ¬corrupted_main[round[act]][2] )
        next(decide[round[act]][act]) := 1;
    next(decide_value[round[act]][act]) := 1;
    for (c = 0; c ≤ 1; c = c + 1)
        next(pre_votes[round[act]] + 1)[c][1] := pre_votes[round[act]] + 1)[c][1] = pre_votes[round[act]] + 1)[c][1] + 1;
        /* may decide on 0 and must cast a pre-vote for 0 */
    else if (main_votes[round[act]][0] = M ∧ corrupted_main[round[act]][0] ∧
        decide[round[act]][act] = 0 ∧ ¬corrupted_main[round[act]][1] )
        next(decide[round[act]][act]) := 0, 1;
    next(decide_value[round[act]][act]) := 0;
    for (c = 0; c ≤ 1; c = c + 1)
next(pre_votes[round[act] + 1][c][0]) :=

/* may decide on 1 and must cast a pre-vote for 1 */
else if (main_votes[round[act]][1] = M ∧ corrupted_main[round[act]][1] ∧
  main_votes[round[act]][0] = 0 ∧ ¬corrupted_main[round[act]][0]) {
  next(decide[round[act]][act]) := {0, 1};
  next(decide_value[round[act]][act]) := 1;
  for (c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act] + 1][c][1]) :=
}

/* may decide on 0 and can cast a pre-vote for 0 or 1 */
else if (main_votes[round[act]][0] = M ∧ corrupted_main[round[act]][0]) {
  next(decide[round[act]][act]) := {0, 1};
  next(decide_value[round[act]][act]) := 0;
  for (c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act] + 1][c][0]) :=
}

/* cannot decide and must cast a pre-vote for 0 */
else if ((main_votes[round[act]][0] > 0 ∨ corrupted_main[round[act]][0]) ∧
  main_votes[round[act]][1] = 0 ∧
  ¬corrupted_main[round[act]][1] ∧
  (main_votes[round[act]][2] < M ∨ ¬corrupted_main[round[act]][2])) {
  for (c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act] + 1][c][0]) :=
}

/* cannot decide and must cast a pre-vote for 1 */
else if ((main_votes[round[act]][1] > 0 ∨ corrupted_main[round[act]][1]) ∧
  main_votes[round[act]][0] = 0 ∧
  ¬corrupted_main[round[act]][0] ∧
  (main_votes[round[act]][2] < M ∨ ¬corrupted_main[round[act]][2])) {
  for (c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act] + 1][c][1]) :=
}

/* cannot decide and may cast a pre-vote for 0 or 1 */
else if ((main_votes[round[act]][0] > 0 ∨ corrupted_main[round[act]][0]) ∧
  (main_votes[round[act]][1] > 0 ∨ corrupted_main[round[act]][1]) ∧
  (main_votes[round[act]][2] < M ∨ ¬corrupted_main[round[act]][2])) {
  for (c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act] + 1][c][1]) :=
}

/* cannot decide and may vote for 0, 1 or coin */
else if ((main_votes[round[act]][0] > 0 ∨ corrupted_main[round[act]][0]) ∧
  (main_votes[round[act]][1] > 0 ∨ corrupted_main[round[act]][1]) ∧
  (main_votes[round[act]][2] < M ∨ ¬corrupted_main[round[act]][2])) {
  if (nd_pre3 = 0) { /* concrete vote for 0 */
    for (c = 0; c ≤ 1; c = c + 1)
      next(pre_votes[round[act] + 1][c][0]) :=
  }
  else if (nd_pre3 = 1) { /* concrete vote for 1 */
    for (c = 0; c ≤ 1; c = c + 1)
      next(pre_votes[round[act] + 1][c][1]) :=
  }
}

next(pre_votes[round[act]+1][c][0]) :=
    pre_votes[round[act]+1][c][0] \cap M \Rightarrow pre_votes[round[act]+1][c][0] := pre_votes[round[act]+1][c][0]+1;

} else { /* coin */
    for(c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act]+1][c][c]) :=
        pre_votes[round[act]+1][c][c] \cap M \Rightarrow pre_votes[round[act]+1][c][c] := pre_votes[round[act]+1][c][c]+1;

} /* cannot decide and may vote for 0 or coin */
else if (main_votes[round[act]][0] > 0 \lor corrupted_main[round[act]][0]) {
    if (nd_pre1=0) { /* concrete vote for 0 */
        for(c = 0; c ≤ 1; c = c + 1)
        next(pre_votes[round[act]+1][c][0]) :=
            pre_votes[round[act]+1][c][0] \cap M \Rightarrow pre_votes[round[act]+1][c][0] := pre_votes[round[act]+1][c][0]+1;

} else { /* coin */
    for(c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act]+1][c][c]) :=
        pre_votes[round[act]+1][c][c] \cap M \Rightarrow pre_votes[round[act]+1][c][c] := pre_votes[round[act]+1][c][c]+1;

} /* cannot decide and may vote for 1 or coin */
else if (main_votes[round[act]][1] > 0 \lor corrupted_main[round[act]][1]) {
    if (nd_pre2=1) { /* concrete vote for 1 */
        for(c = 0; c ≤ 1; c = c + 1)
        next(pre_votes[round[act]+1][c][1]) :=
            pre_votes[round[act]+1][c][1] \cap M \Rightarrow pre_votes[round[act]+1][c][1] := pre_votes[round[act]+1][c][1]+1;

} else { /* coin */
    for(c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act]+1][c][c]) :=
        pre_votes[round[act]+1][c][c] \cap M \Rightarrow pre_votes[round[act]+1][c][c] := pre_votes[round[act]+1][c][c]+1;

} else { /* left with cannot decide and vote for coin */
    for(c = 0; c ≤ 1; c = c + 1)
    next(pre_votes[round[act]+1][c][c]) :=
        pre_votes[round[act]+1][c][c] \cap M \Rightarrow pre_votes[round[act]+1][c][c] := pre_votes[round[act]+1][c][c]+1;

} }

COIN:
/* if decided in previous round then done else continue */
ext(pc[act]) := (round[act] > 0 \land \text{decide}[round(\text{act})-1][\text{act}]) \Rightarrow \text{DONE} := \text{MAIN};
next(round[act]) := (round[act] > 0 \land \text{decide}[round(\text{act})-1][\text{act}]) \Rightarrow round[act] := round[act]+1; /* increase round */
/* toss coin if at least M honest parties have reached this stage and the coin has not been tossed */
ext(coin[round[act]+1]) := pre[round[act]+1] = M \land \text{coin}[round[act]+1] = \text{not_tossed} \Rightarrow \{0,1\} := \text{coin}[round[act]+1];

} /* corrupted_pre[r][c][v] is true if a corrupted party can have a pre vote for v in round r given the coin equals c */
corrupted_pre : array ROUNDS of array 0..1 of array 0..1 of boolean;
/* corrupted_main[r][v] is true if a corrupted party can have a main vote for v in round r */
corrupted_main : array ROUNDS of array 0..2 of boolean;
/* INITIAL VALUES */
/* initially cannot vote for either */
forall(r in ROUNDS)
{
    for(c = 0; c <= 1; c = c + 1) for(v = 0; v <= 1; v = v + 1) init(corrupted_pre[r][c][v]) := 0;
    for(v = 0; v <= 2; v = v + 1) init(corrupted_main[r][v]) := 0;
}

/* NEXT VALUES */
/* pre votes: for round 0 need at least one preprocessing vote from an honest party for a value to vote for that value */
/* in general to vote v either need a main vote for v (from either an honest or corrupted party) in the previous round */
/* or is able to vote for the coin, that is, there are need sufficient abstain main votes in the previous round */
forall (r in ROUNDS) 
{
    if (r=0) 
    {
        for(v = 0; v <= 1; v = v + 1) next(corrupted_pre[r][0][v]) := (next(pre_proc_votes[v])>0);
        for(v = 0; v <= 1; v = v + 1) next(corrupted_pre[r][1][v]) := (next(pre_proc_votes[v])>0);
    } 
    else 
    {
        next(corrupted_pre[r][0][0]) :=
        (next(main[r-1])=M ∧ (next(main_votes[r-1][0])>0 ∨ next(corrupted_main[r-1][0])) ∨ 
        (next(main_votes[r-1][2])=M ∧ next(corrupted_main[r-1][2]))) ? 1 : 0;
        next(corrupted_pre[r][0][1]) :=
        (next(main[r-1])=M ∧ (next(main_votes[r-1][1])>0 ∨ next(corrupted_main[r-1][1])) ? 1 : 0;
        next(corrupted_pre[r][1][0]) :=
        (next(main[r-1])=M ∧ (next(main_votes[r-1][0])>0 ∨ next(corrupted_main[r-1][0])) ? 1 : 0;
        next(corrupted_pre[r][1][1]) :=
        (next(main[r-1])=M ∧ (next(main_votes[r-1][1])>0 ∨ next(corrupted_main[r-1][1])) ∨ 
        (next(main_votes[r-1][2])=M ∧ next(corrupted_main[r-1][2])) ? 1 : 0;
    }

/* main votes: to vote for v (0 or 1) need M pre votes for v from honest parties also pre votes for v from corrupted parties */
/* to vote for abstain need a pre vote for 0 and a pre vote for 1 (from either honest or corrupted parties) */
forall (r in ROUNDS) 
{
    if (next(coin[r])=0) 
    {
        for(v = 0; v <= 1; v = v + 1) next(corrupted_main[r][v]) :=
        (next(pre[r])=M ∧ next(pre_votes[r][0][v])=M ∧ next(corrupted_pre[r][0][v]) ) ? 1 : 0;
        next(corrupted_main[r][2]) :=
        ( next(pre[r])=M ∧ (next(pre_votes[r][0][0])>0 ∨ next(corrupted_pre[r][0][0])) ∧ 
        (next(pre_votes[r][0][1])>0 ∨ next(corrupted_pre[r][0][1]))) ? 1 : 0;
    } 
    else if (next(coin[r])=1) 
    {
        for(v = 0; v <= 1; v = v + 1) next(corrupted_main[r][v]) :=
        (next(pre[r])=M ∧ next(pre_votes[r][1][v])=M ∧ next(corrupted_pre[r][1][v]) ) ? 1 : 0;
        next(corrupted_main[r][2]) :=
        ( next(pre[r])=M ∧ (next(pre_votes[r][1][0])>0 ∨ next(corrupted_pre[r][1][0])) ∧ 
        (next(pre_votes[r][1][1])>0 ∨ next(corrupted_pre[r][1][1]))) ? 1 : 0;
    } 
}

/* -----------END OF PROTOCOL------------- */

/* ----------------HISTORY VARIABLES---------------- */
/* for each round records the first party to have a main-vote for each value */
history_main_votes : array ROUNDS of array 0..2 of PROC;
forall (r in ROUNDS) for(v = 0; v <= 2; v = v + 1) 
{
    init(history_main_votes[r][v]) := act;
    next(history_main_votes[r][v]) := next(main_votes[r][v])>0 ∧ main_votes[r][v]=0 ? act : history_main_votes[r][v];
}
/* for each round records the first party to have a pre-vote for each value */
history_pre_votes : array ROUNDS of array 0..1 of array 0..1 of PROC;
forall (r in ROUNDS) for(c = 0; c <= 1; c = c + 1) for(v = 0; v <= 1; v = v + 1) 
{
\[
\text{init}(\text{history\_pre\_votes}[r][c][v]) := \text{act};
\]
\[
\text{next}(\text{history\_pre\_votes}[r][c][v]) := \text{next}(\text{pre\_votes}[r][c][v]) > 0 \land \text{pre\_votes}[r][c][v] = 0 \ ? \ \text{act} : \text{history\_pre\_votes}[r][c][v];
\]

/* for each round records the first party to have a pre-vote */
\text{history\_pre} : \text{array} \text{ROUNDS of} \text{PROC};
\forall (r \in \text{ROUNDS}) \{
\text{init}(\text{history\_pre}[r]) := \text{act};
\text{next}(\text{history\_pre}[r]) := \text{next}(\text{pre}[r]) > 0 \land \text{pre}[r] = 0 \ ? \ \text{act} : \text{history\_pre}[r];
\}

/* records the first party to have a pre-processing-vote for each value */
\text{history\_pre\_proc\_votes} : \text{array} 0..1\ of\ \text{PROC};
\text{for}(v = 0; v \leq 1; v = v + 1) \{
\text{init}(\text{history\_pre\_proc\_votes}[v]) := \text{act};
\text{next}(\text{history\_pre\_proc\_votes}[v]) := \text{next}(\text{pre\_proc\_votes}[v]) > 0 \land \text{pre\_proc\_votes}[v] = 0 \ ? \ \text{act} : \text{history\_pre\_proc\_votes}[v];
\}

/* records the party which flips the coin */
\text{history\_coin} : \text{array} \text{ROUNDS of} \text{PROC};
\forall (r \in \text{ROUNDS}) \{
\text{init}(\text{history\_coin}[r]) := \text{act};
\text{next}(\text{history\_coin}[r]) := !\text{next}(\text{coin}[r]) = \text{not\_tossed} \land \text{coin}[r] = \text{not\_tossed} \ ? \ \text{act} : \text{history\_coin}[r];
\}