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ESSAI & ACAI 2023
LJUBLJANA, SLOVENIA

MODEL UNCERTAINTY IN SEQUENTIAL DECISION MAKING

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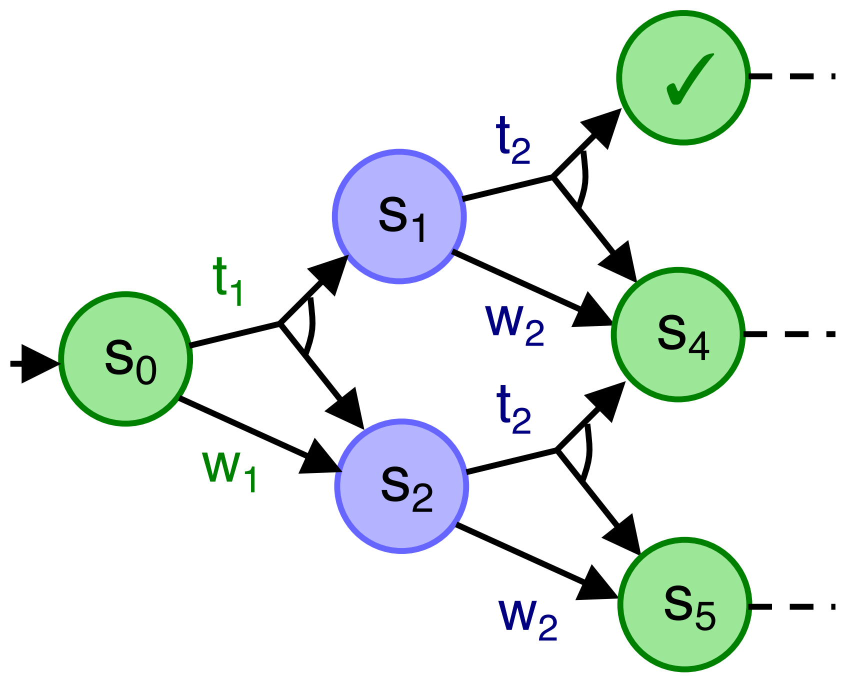
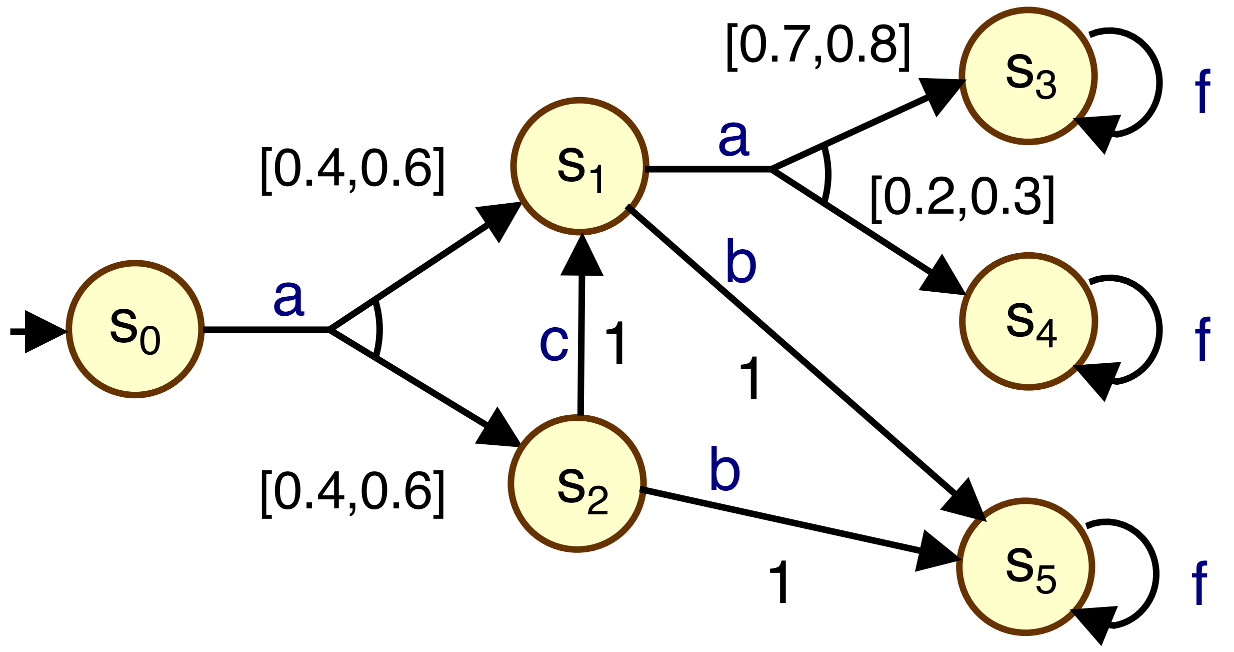
Recap

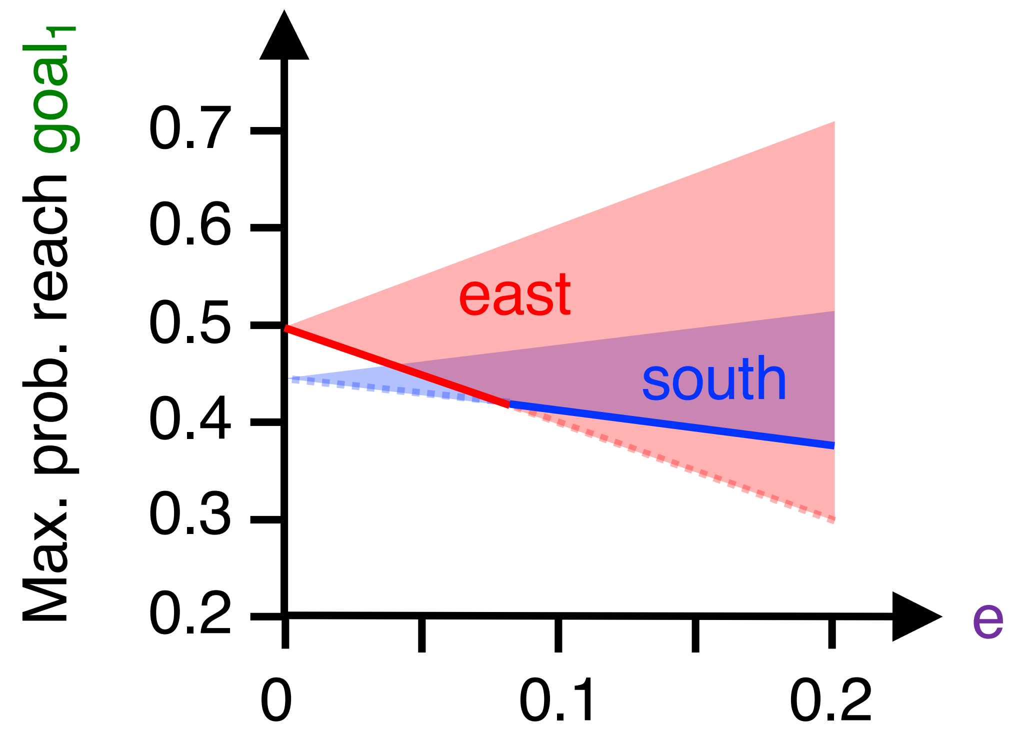
- Stochastic games
  - unknown parts of the system can be modelled adversarially
  - zero-sum turn-based (or concurrent) stochastic games
    - dynamic programming (value iteration) generalises



- Uncertain MDPs
  - MDPs plus epistemic uncertainty: set of transition functions
    - each $P \in \mathscr{P}$ is a transition function $P : S \times A \times S \rightarrow [0,1]$
  - rectangularity (dependencies)
  - control policies + robust control
  - environment policies

$$V^*(s) = \max_{\pi \in \Pi} \min_{P \in \mathscr{P}} V^{\pi,P}(s)$$





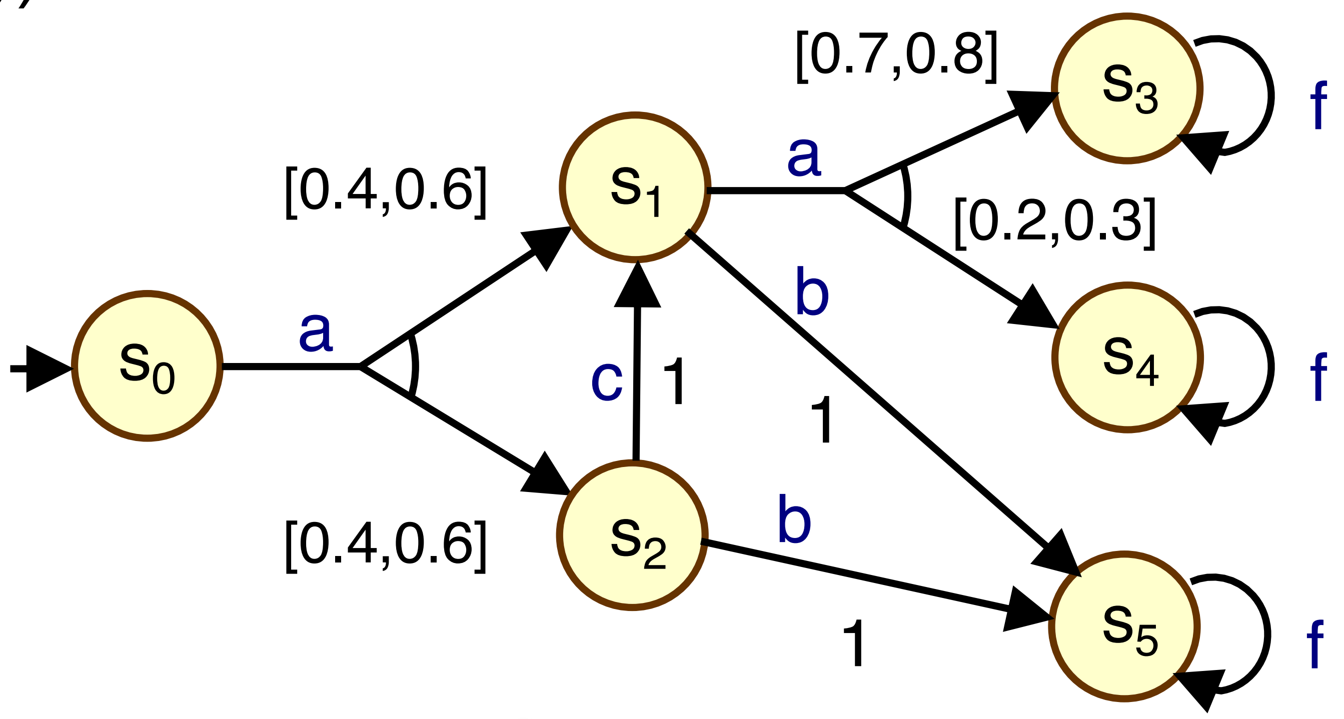
Course contents

- ~~Markov decision processes (MDPs) and stochastic games~~
  - ~~MDPs: key concepts and algorithms~~
  - ~~stochastic games: adding adversarial aspects~~
- Uncertain MDPs
  - ~~MDPs + epistemic uncertainty, robust control,~~ robust dynamic programming, interval MDPs, uncertainty set representation, challenges, tools
- Sampling-based uncertain MDPs
  - removing the transition independence assumption
- Bayes-adaptive MDPs
  - maintaining a distribution over the possible models

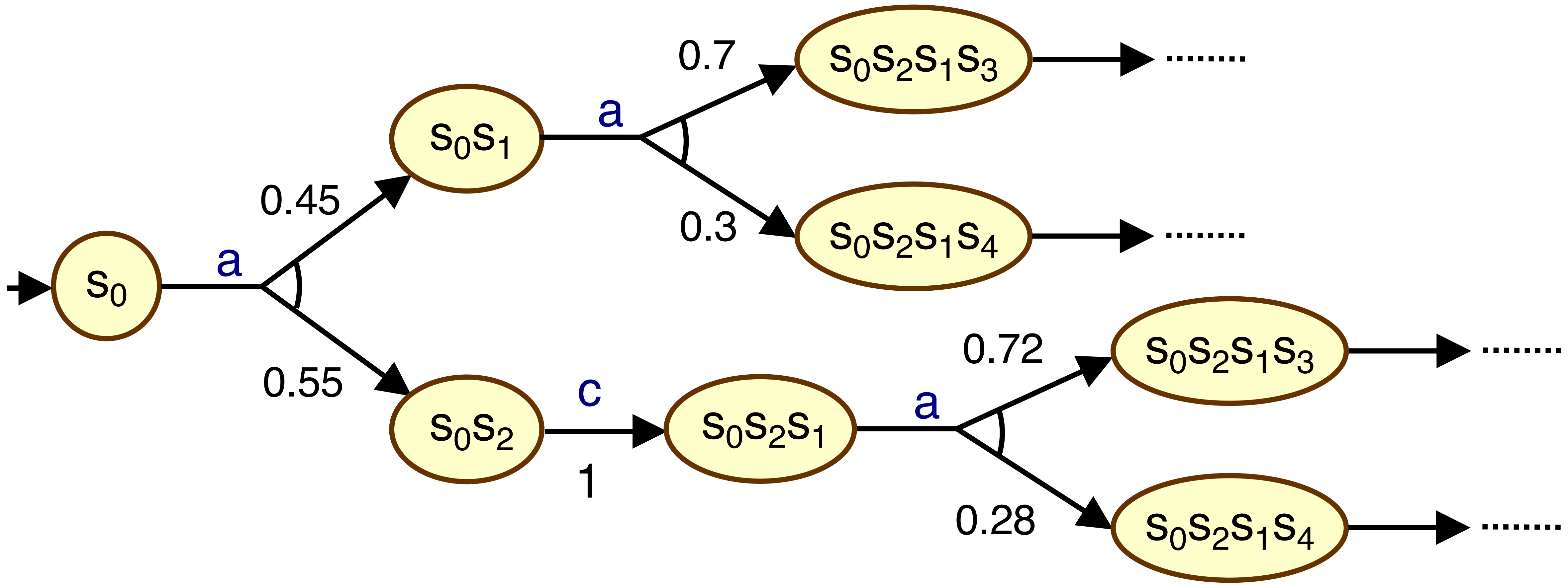
Robust control

Resolving uncertainty

- Now we consider a more dynamic approach to resolving uncertainty
  - (which we will need to extend dynamic programming to this setting)
- An environment policy (or nature policy, or adversary) $\tau \in \mathcal{T}$
  - is a mapping $\tau : (S \times A)^* \times (S \times A) \rightarrow Dist(S)$
  - such that $\tau(s_0, a_0, \ldots, s_n, a_n) \in \mathcal{P}_s^a$
  - note: this assumes (s,a)-rectangularity!

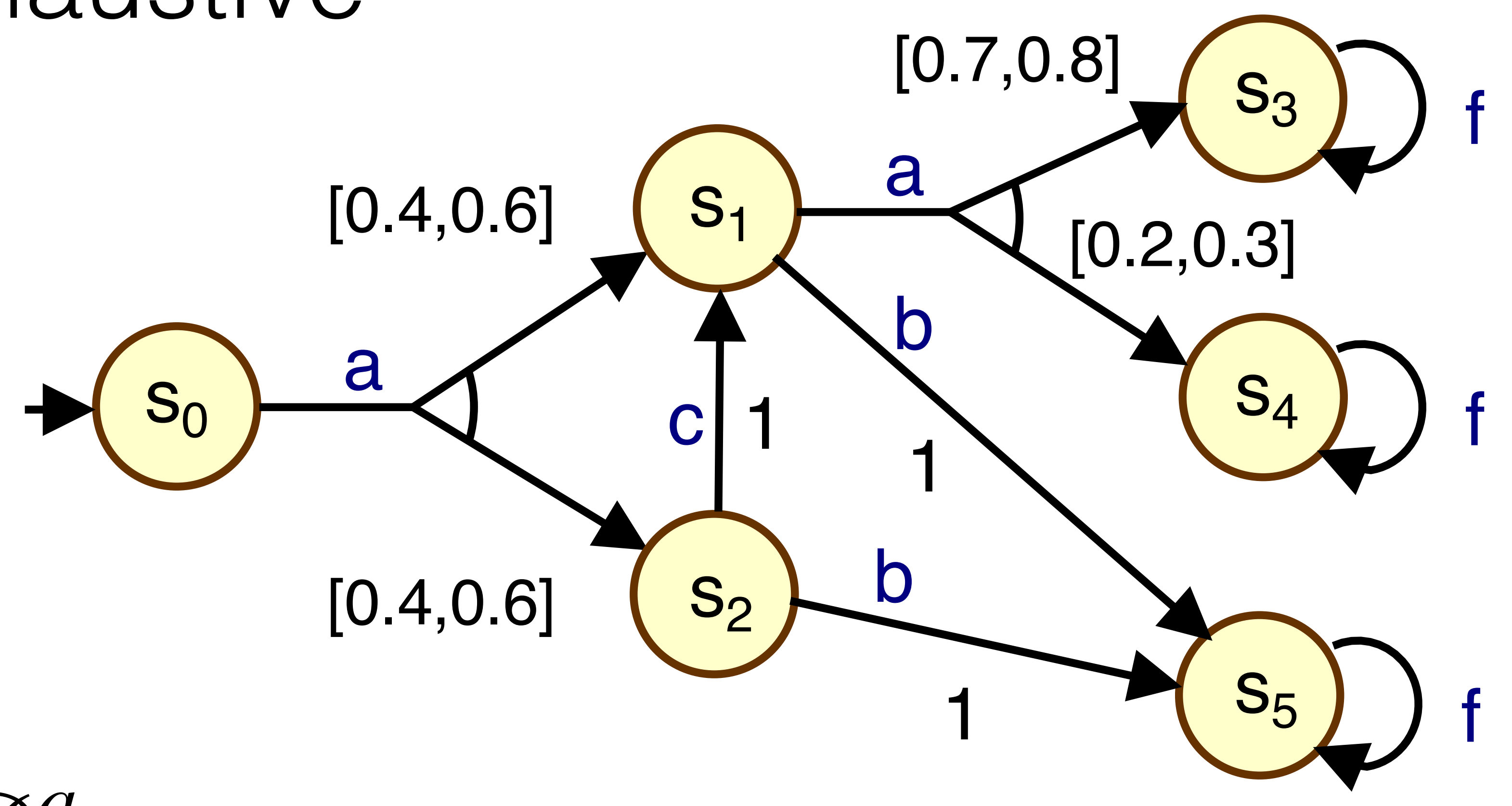


- Policies π, τ yield
  - a probability space $Pr_s^{\pi,\tau}$
  - random variables $\mathbb{E}_s^{\pi,\tau}(X)$
  - and value functions $V^{\pi,\tau}$



Dynamic vs. static uncertainty

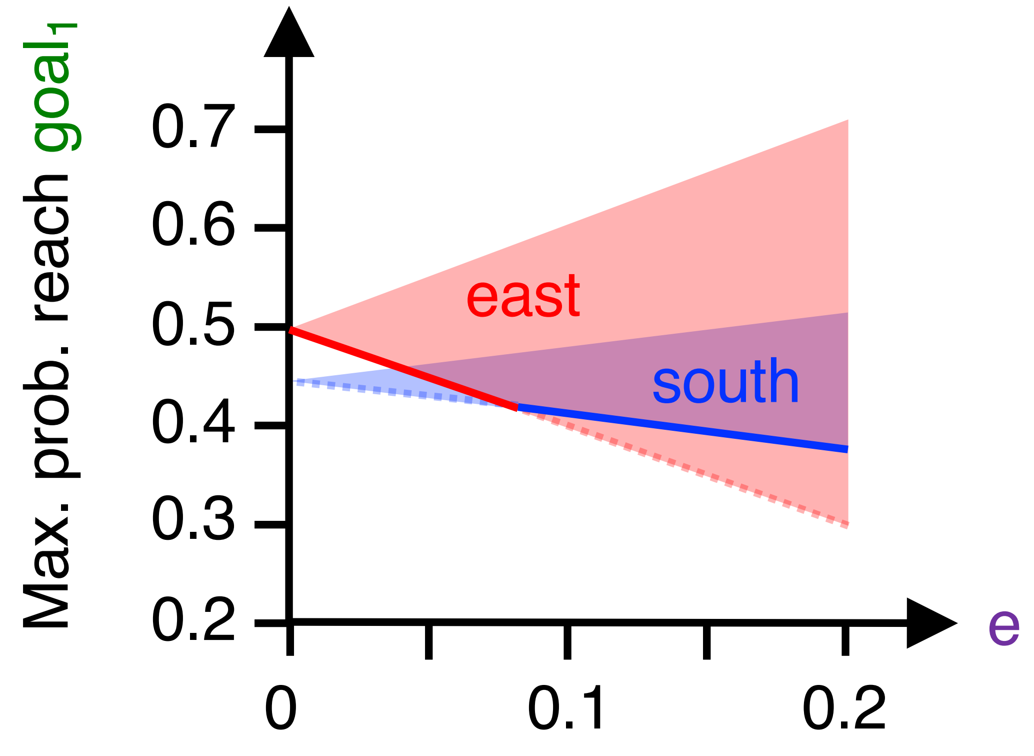
- Quantifying over environment policies $\tau \in \mathscr{T}$ is more exhaustive
  - than quantifying over transition probabilities $P \in \mathscr{P}$
  - $\{Pr_s^{\pi,P} : P \in \mathscr{P}\} \subseteq \{Pr_s^{\pi,\tau} : \tau \in \mathscr{T}\}$



- Memoryless (stationary) environment policies $\tau_m \in \mathscr{T}_m$
  - are mappings $\tau_m : S \times A \to Dist(S)$ such that $\tau_m(s,a) \in \mathscr{P}_s^a$
  - in this case, the semantics now coincide:
  - $\{Pr_s^{\pi,P} : P \in \mathscr{P}\} = \{Pr_s^{\pi,\tau_m} : \tau_m \in \mathscr{T}_m\}$

- We call this dynamic uncertainty ($\tau \in \mathscr{T}$) vs. static uncertainty ($P \in \mathscr{P}$)
  - which to use is a modelling decision (e.g., on the timing of events)
  - but there are also implications for tractability
  - similar situation to rectangularity (uncertainty set independence)

Robust control (revisited)



- Robust control
  - but quantifying over policies (rather than uncertainty sets)

- Now we have
  - optimal worst-case value

$$V^*(s) = V^{\Pi,\mathcal{T}}(s) = \max_{\pi\in\Pi} \min_{\tau\in\mathcal{T}} V^{\pi,\tau}(s)$$

notation for optimal value for sets of control/environment policy sets $\Pi, \mathcal{T}$

  - optimal worst-case policy

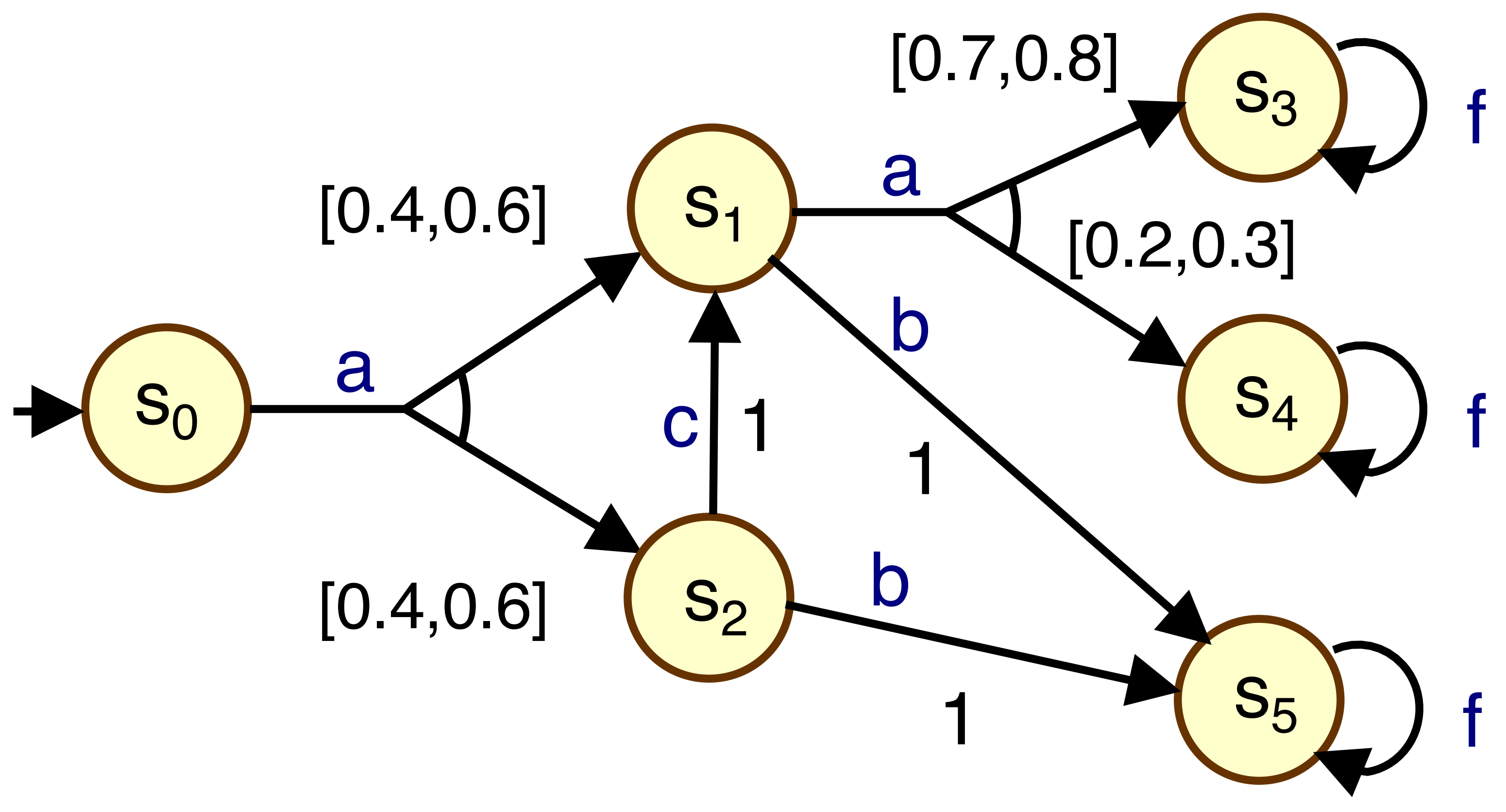
$$\pi^* = \operatorname*{argmax}_{\pi\in\Pi} \min_{\tau\in\mathcal{T}} V^{\pi,\tau}(s)$$

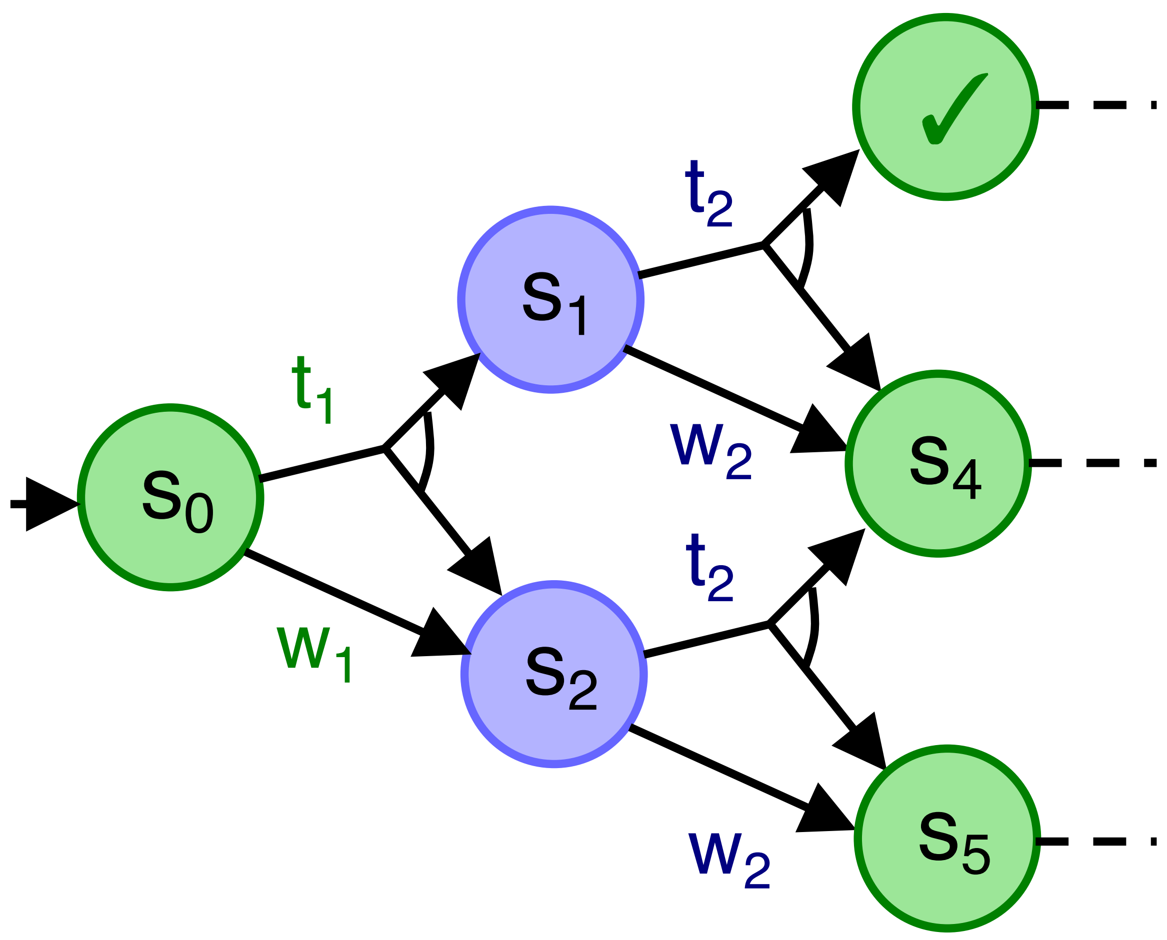
- Note that we may want to quantify over mismatching sets of policies, e.g.:

$$V^{\Pi,\mathcal{T}_m}(s) = \max_{\pi\in\Pi} \min_{\tau_m\in\mathcal{T}_m} V^{\pi,\tau_m}(s) = \max_{\pi\in\Pi} \min_{P\in\mathcal{P}} V^{\pi,P}(s)$$

e.g. for static uncertainty

uMDPs vs stochastic games





Robust dynamic programming

- Let's again focus on optimising MaxProb (the situation is similar for SSP)
  - and recall: we need to assume (s,a)-rectangularity
- Memoryless policies suffice, for both the controller and the environment

$$V^{\Pi,\mathscr{T}}(s_0) = V^{\Pi_m,\mathscr{T}_m}(s_0) = V^{\Pi_m,\mathscr{T}}(s_0) = V^{\Pi,\mathscr{T}_m}(s_0)$$

- Perfect duality:

$$V^{\Pi,\mathscr{T}}(s_0) = \max_{\pi\in\Pi} \min_{\tau\in\mathscr{T}} V^{\pi,\tau}(s_0) = \min_{\tau\in\mathscr{T}} \max_{\pi\in\Pi} V^{\pi,\tau}(s_0)$$

- The optimal value function satisfies the Bellman equation:

$$V^*(s) = V^{\Pi,\mathscr{T}}(s) = \begin{cases} 1 & \text{if } s \in goal \\ \max_{a\in A(s)} \inf_{P_s^a\in\mathscr{P}_s^a} \sum_{s'\in S} P_s^a(s') \cdot V^{\Pi,\mathscr{T}}(s') & \text{otherwise} \end{cases}$$

Robust value iteration

- Optimal values for uMDPs can be obtained using robust value iteration (robust VI)
  - from the limit of the vector sequence defined below
  - $V^*(s) = \lim_{k \to \infty} x_s^k$ where:

We will re-use graph-based pre computation for MDPs

$$x_s^k = \begin{cases} 1 & \text{if } s \in S^1 \\ 0 & \text{if } s \in S^0 \\ 0 & \text{if } s \in S^? \text{ and } k = 0 \\ \max_{a \in A(s)} \inf_{P_s^a \in \mathscr{P}_s^a} \sum_{s' \in S} P_s^a(s') \cdot x_{s'}^{k-1} & \text{otherwise} \end{cases}$$

- Again, this Bellman operator is (i) monotonic (ii) a contraction in the L_∞ norm
  - needs (s-a)-rectangularity, but no assumptions on convexity
  - (it suffices to take convex hull of each $\mathscr{P}_s^a$)

Uncertainty set representations

- The core step of robust VI comprises two nested optimisation problems:

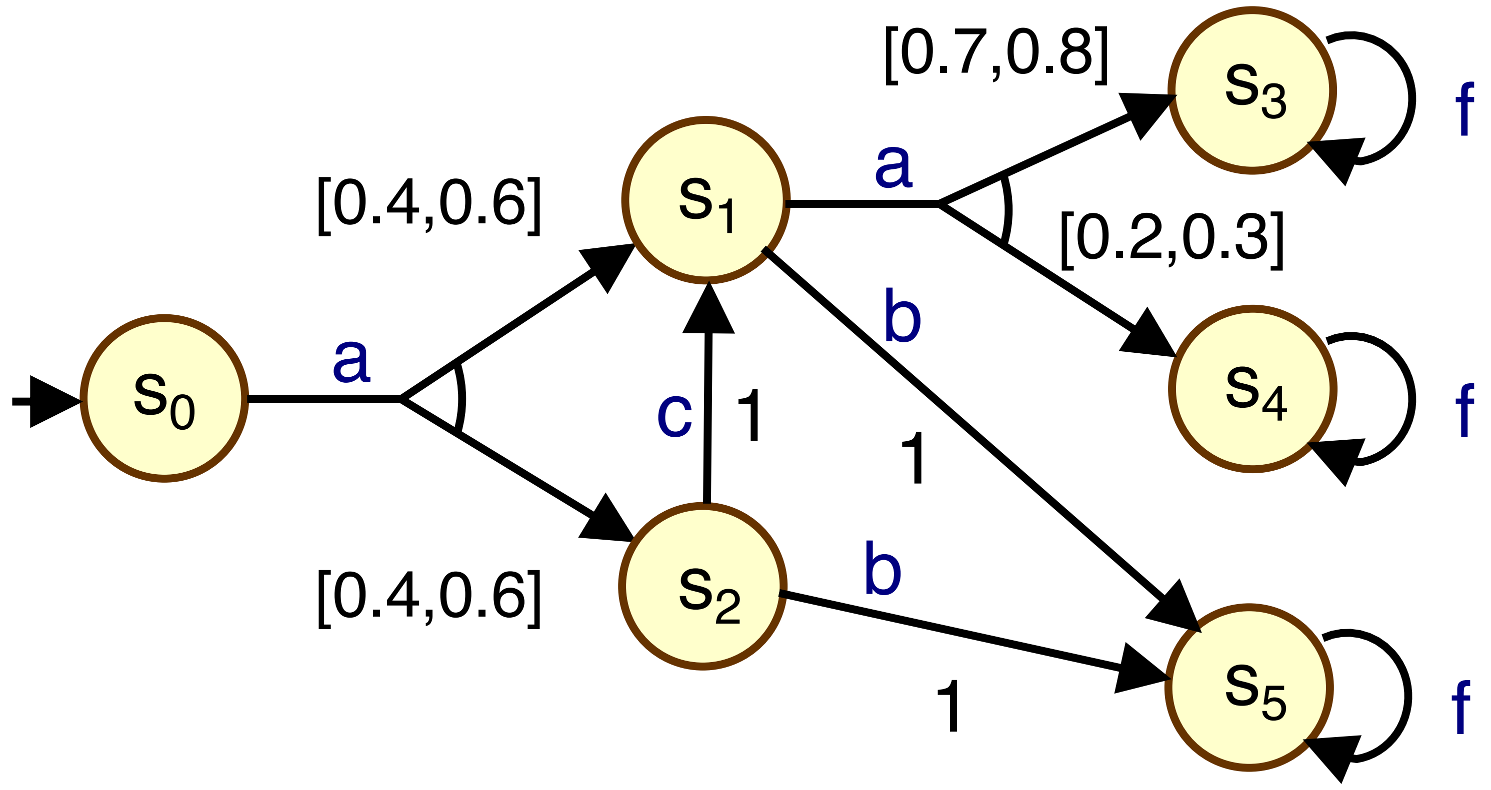
$$\max_{a \in A(s)} \inf_{P_s^a \in \mathscr{P}_s^a} \sum_{s' \in S} P_s^a(s') \cdot x_{s'}$$

  - Outer problem (optimal control action)
  - Inner problem (worst-case transition probabilities)

  where x is some vector of values

- Computational cost: robust VI potentially not much more expensive than VI for MDPs
  - if the inner problem can solved efficiently
  - note: uncertainty sets $\mathscr{P}_s^a$ are usually infinite

- Definition/representation of uncertainty sets?
  - trade off statistical accuracy vs. computation efficiency?

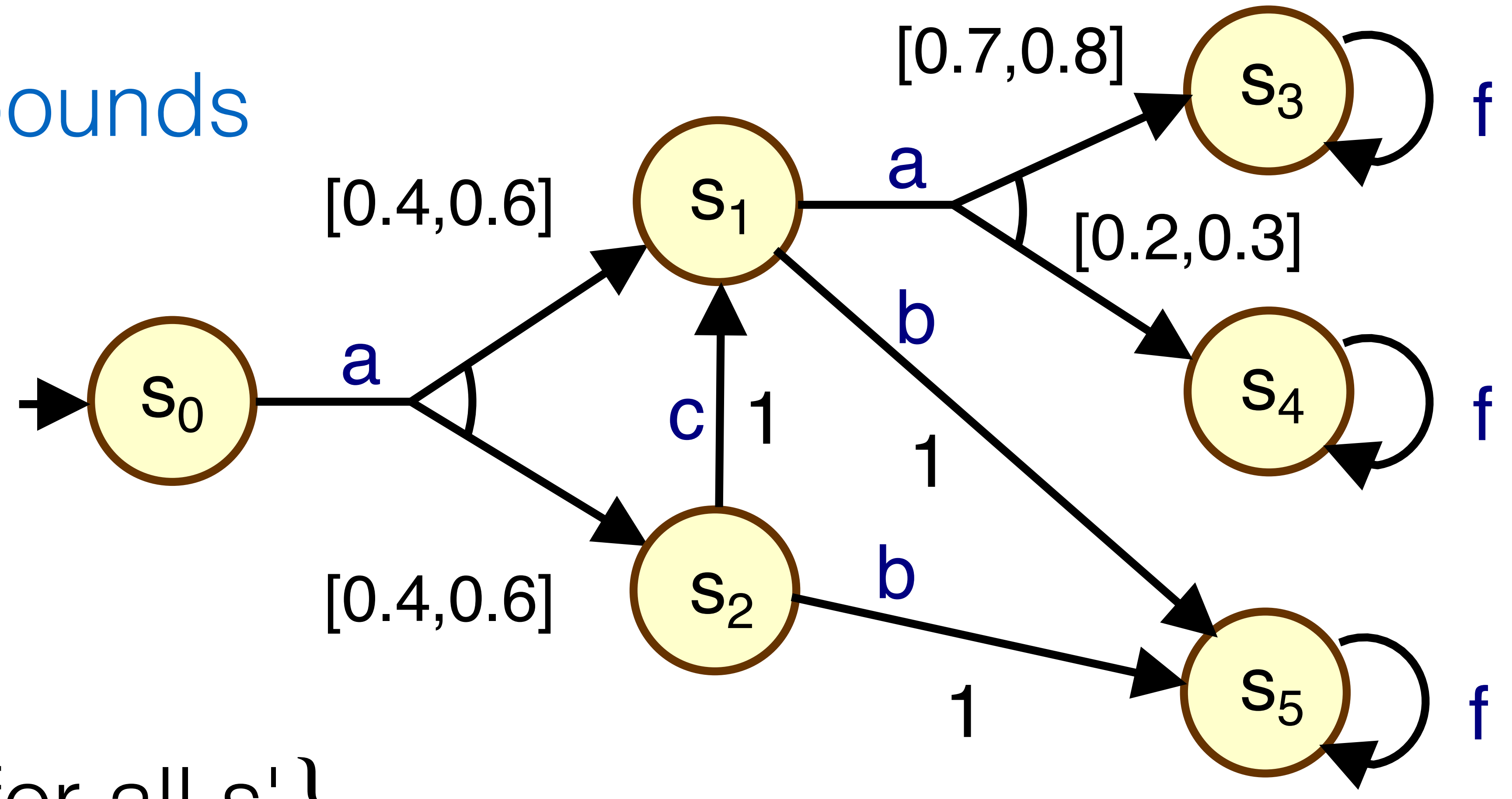


- First example: intervals, a simple uncertainty set representation
  - which suit statistical estimates of confidence intervals for individual transition probabilities

Interval MDPs

Interval MDPs

- An interval MDP (IMDP) is of the form $\mathscr{M} = (S, s_0, A, \underline{P}, \overline{P})$ where:
  - states S, initial state s_0 and actions A are as for MDPs
  - $\underline{P} : S \times A \times S \rightarrow [0,1]$ gives transition probability lower bounds
  - $\overline{P} : S \times A \times S \rightarrow [0,1]$ gives transition probability upper bounds
    - such that $\underline{P}(s,a,s') \leq \overline{P}(s,a,s')$ for all s, a, s'



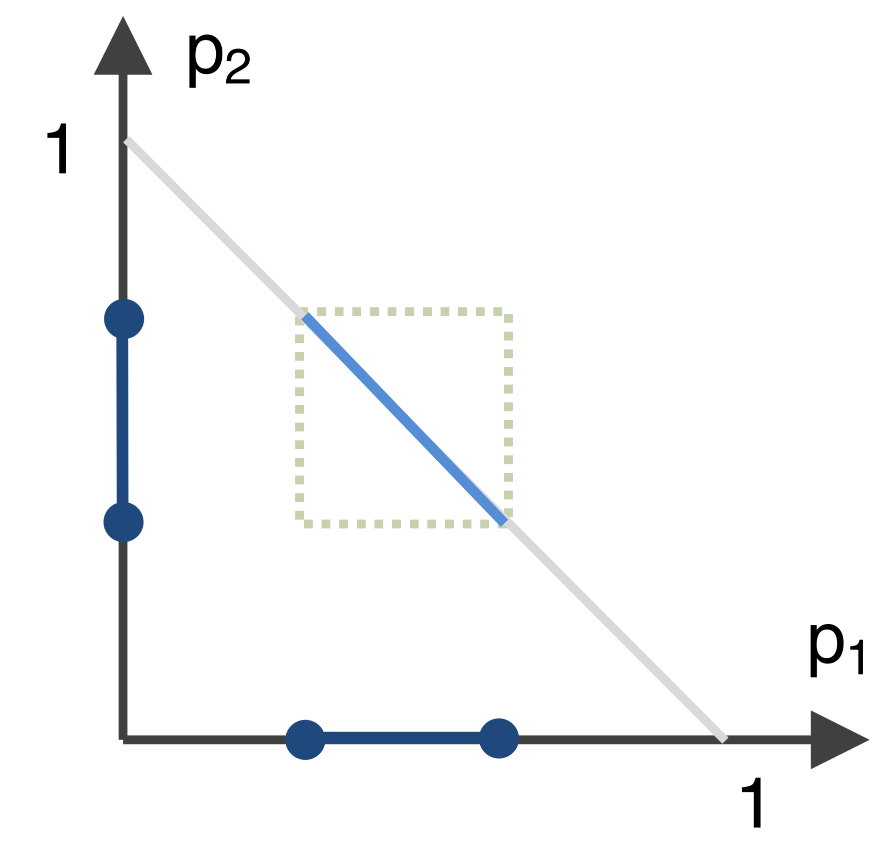
- IMDP uncertainty sets
  - $\mathscr{P}_s^a = \{P_s^a \in Dist(S) \mid \underline{P}(s,a,s') \leq P_s^a(s') \leq \overline{P}(s,a,s')$ for all s'$\}$
    - probabilities are independent (except for the need to sum to 1)
  - $\mathscr{P} = \times_{(s,a)\in S\times A}\, \mathscr{P}_s^a$
    - i.e., IMDPs are (s-a)-rectangular

IMDP uncertainty sets

- Interval uncertainty sets $\mathscr{P}_s^a$ are convex subsets of $[0,1]^{|S|}$

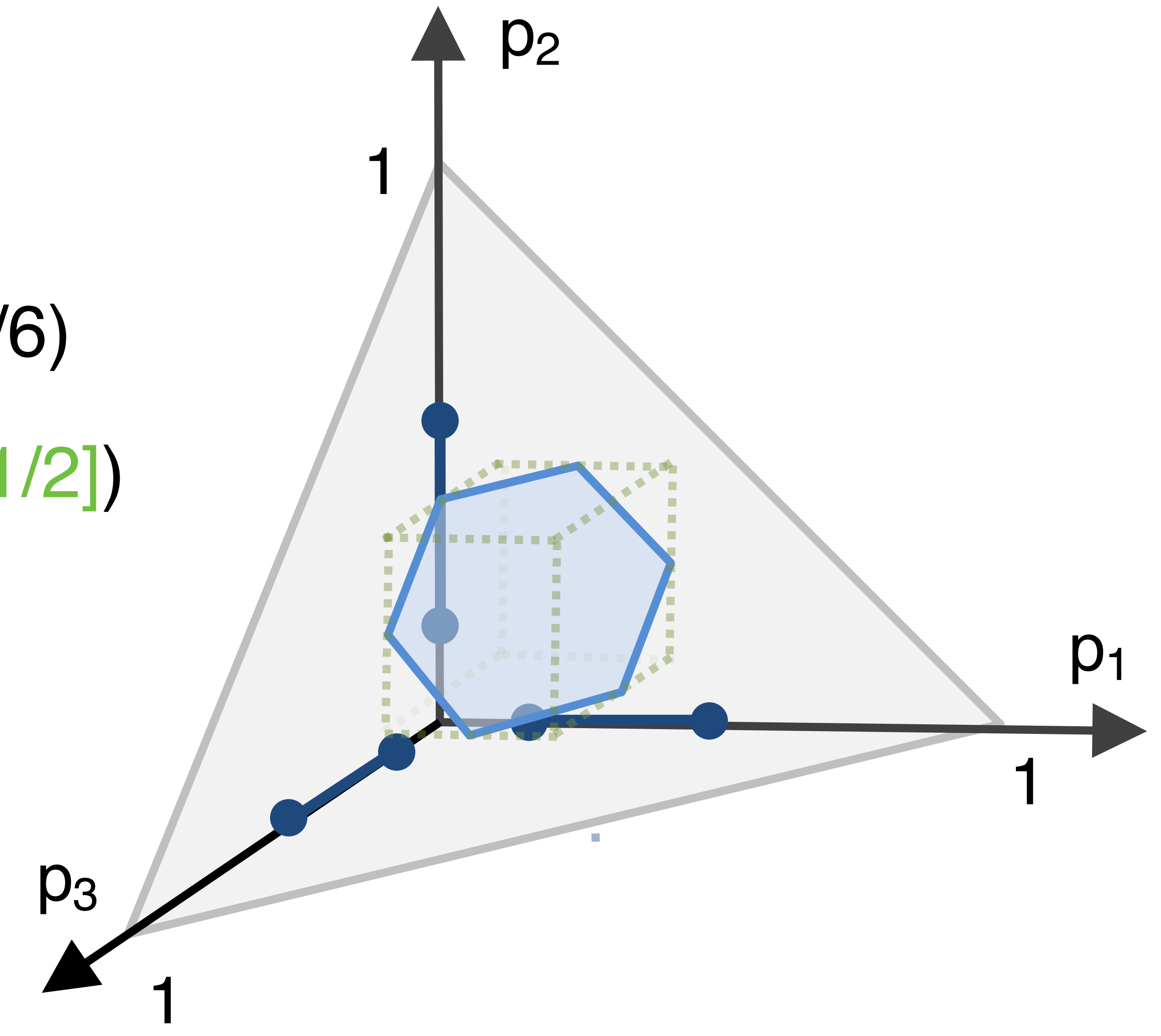
(p1,p2) ∈ (1/2,1/2) ± (1/6,1/6)

= ([1/3,2/3], [1/3,2/3])



(p1,p2,p3) ∈ (1/3,1/3,1/3) ± (1/6,1/6,1/6)

= ([1/6,1/2], [1/6,1/2], [1/6,1/2])



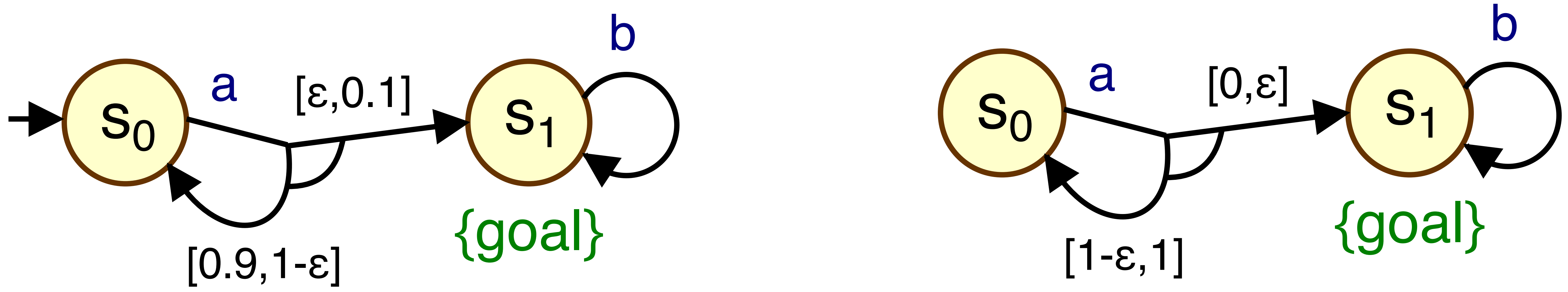
- We can delimit the intervals
  - i.e., trim the interval bounds such that at least one possible distribution takes each extremal value
  - e.g., $\underline{P}(s') := \max[\underline{P}(s'), 1 - \Sigma_{s \neq s'} \overline{P}(s)]$
    - e.g. [0.1,0.4], [0.5,0.8] → [0.2,0.4], [0.6,0.8]

An assumption on IMDPs

- Assumption: IMDPs have a fixed underlying transition graph
  - i.e., for each s,a,s' either: (i) $\underline{P}(s,a,s') > 0$; or

    (ii) $\underline{P}(s,a,s') = \overline{P}(s,a,s') = 0$

- Otherwise behaviour can be qualitatively different for small changes in $P(s,a,s')$



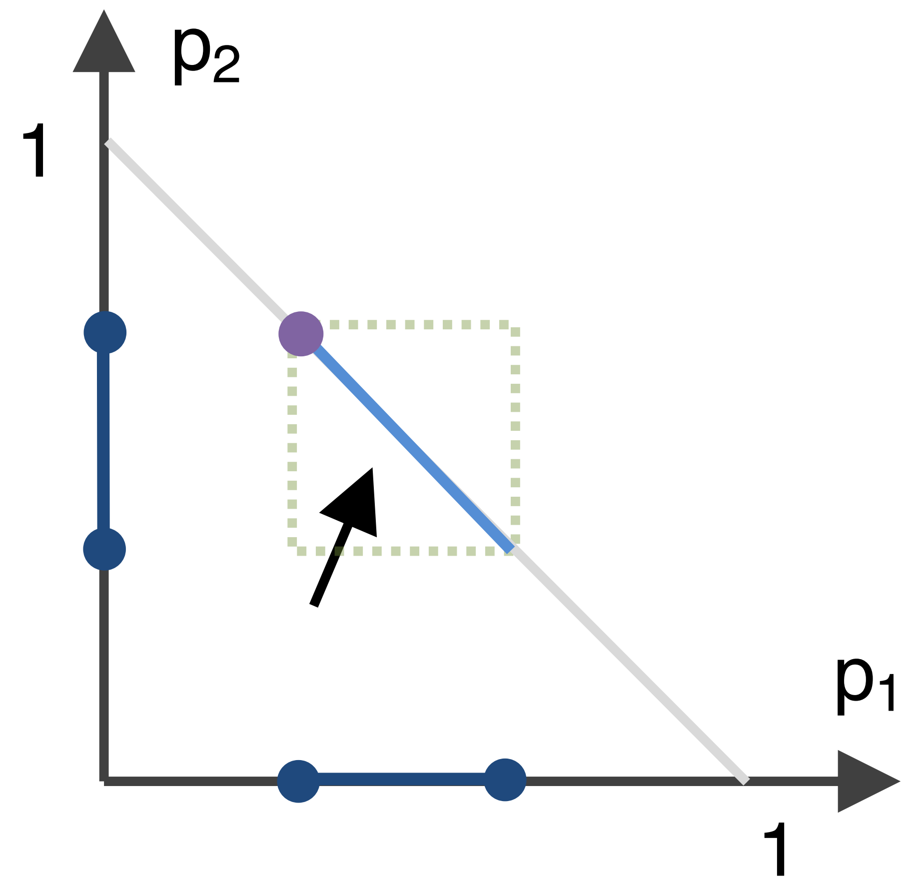
  - For $\varepsilon > 0$, the probability to reach goal is always 1
  - For $\varepsilon = 0$, the probability to reach goal can be 0
  - (contrast to, e.g., a finite-horizon property $\text{MaxProb}^{\leq k}$(goal))

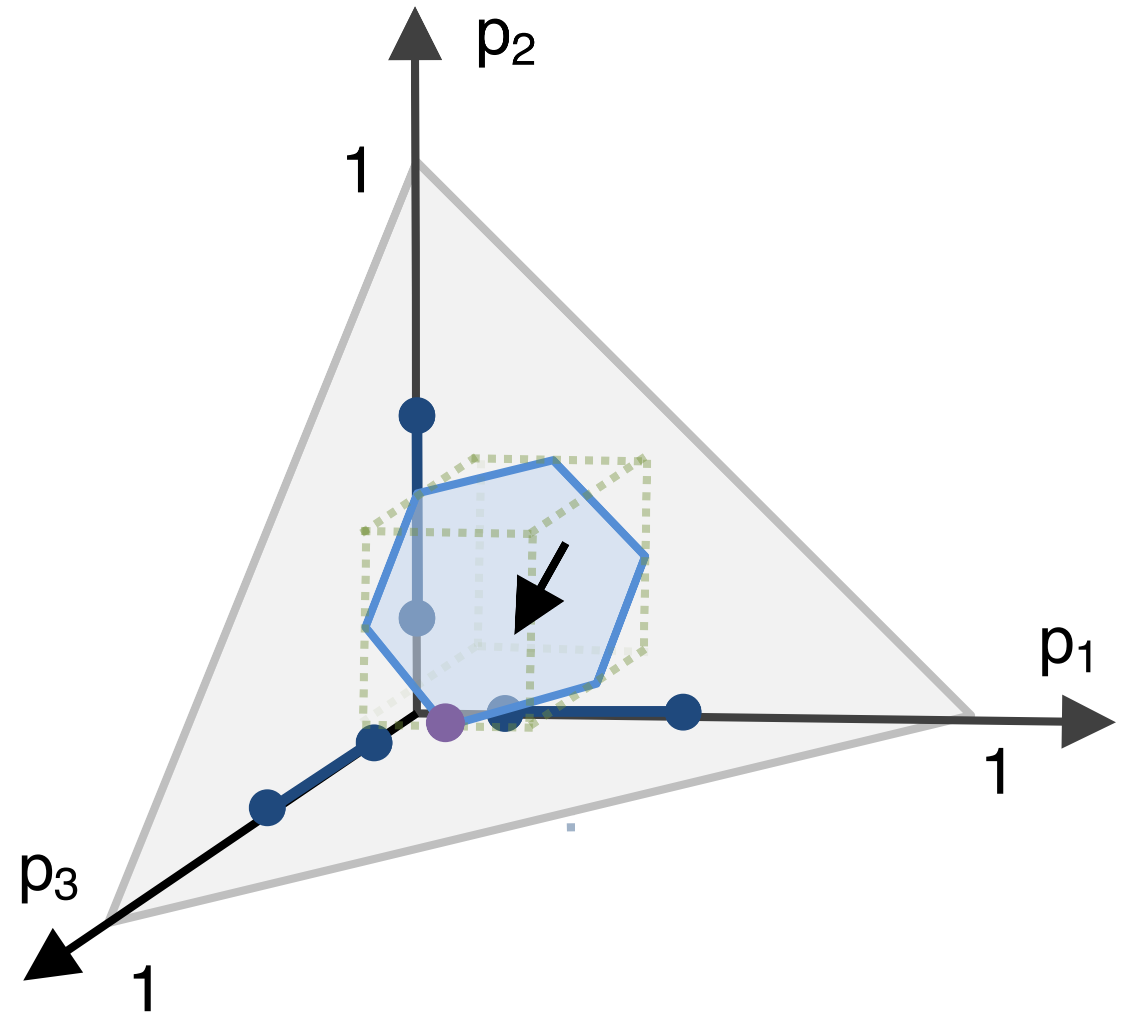
Robust value iteration for IMDPs

- The inner problem for each iteration, and each (s, a) is: $\inf_{P_s^a \in \mathscr{P}_s^a} \sum_{s' \in S} P_s^a(s') \cdot x_{s'}$

- Can be solved via a linear programming problem:
  - let $p_{s'}$ be $|S|$ variables for the chosen probabilities $P_s^a(s')$

> minimise $\Sigma_{s'}\, p_{s'} \cdot x_{s'}$ such that:
>
> $\underline{P}_s^a(s') \leq p_{s'} \leq \overline{P}_s^a(s')$ for all s' and $\Sigma_{s'} p_{s'} = 1$

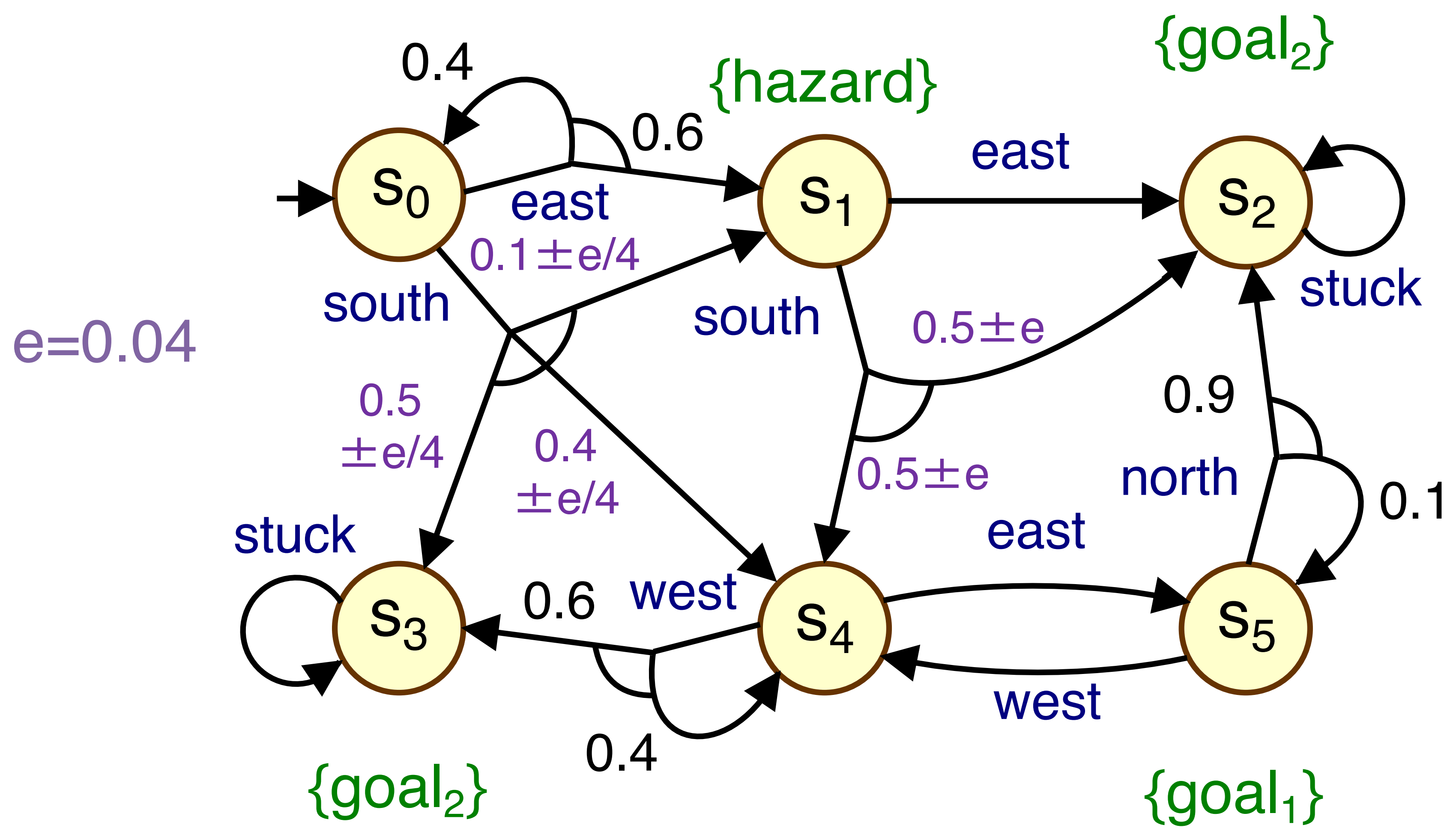




- We can also solve this more directly by sorting
  - sort the values $x_{s'}$ into ascending order
  - for increasing values x_{s_i} assign the maximum possible value to p_{s_i}
  - which is bounded by 1 - (the sum of actual/min values for other p_{s_j})

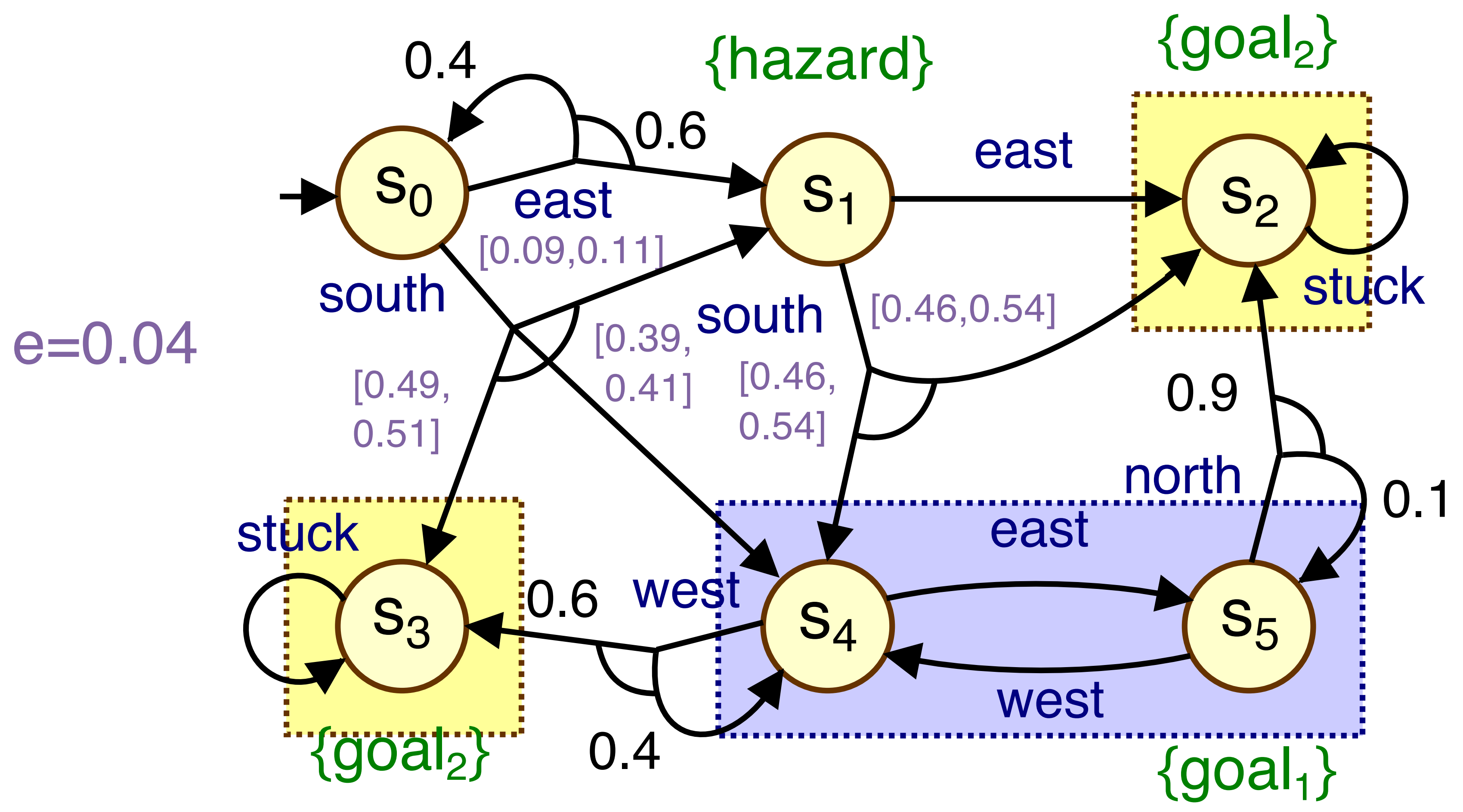
Running example: IMDPs and robust VI

- Example: MaxProb($goal_1$)



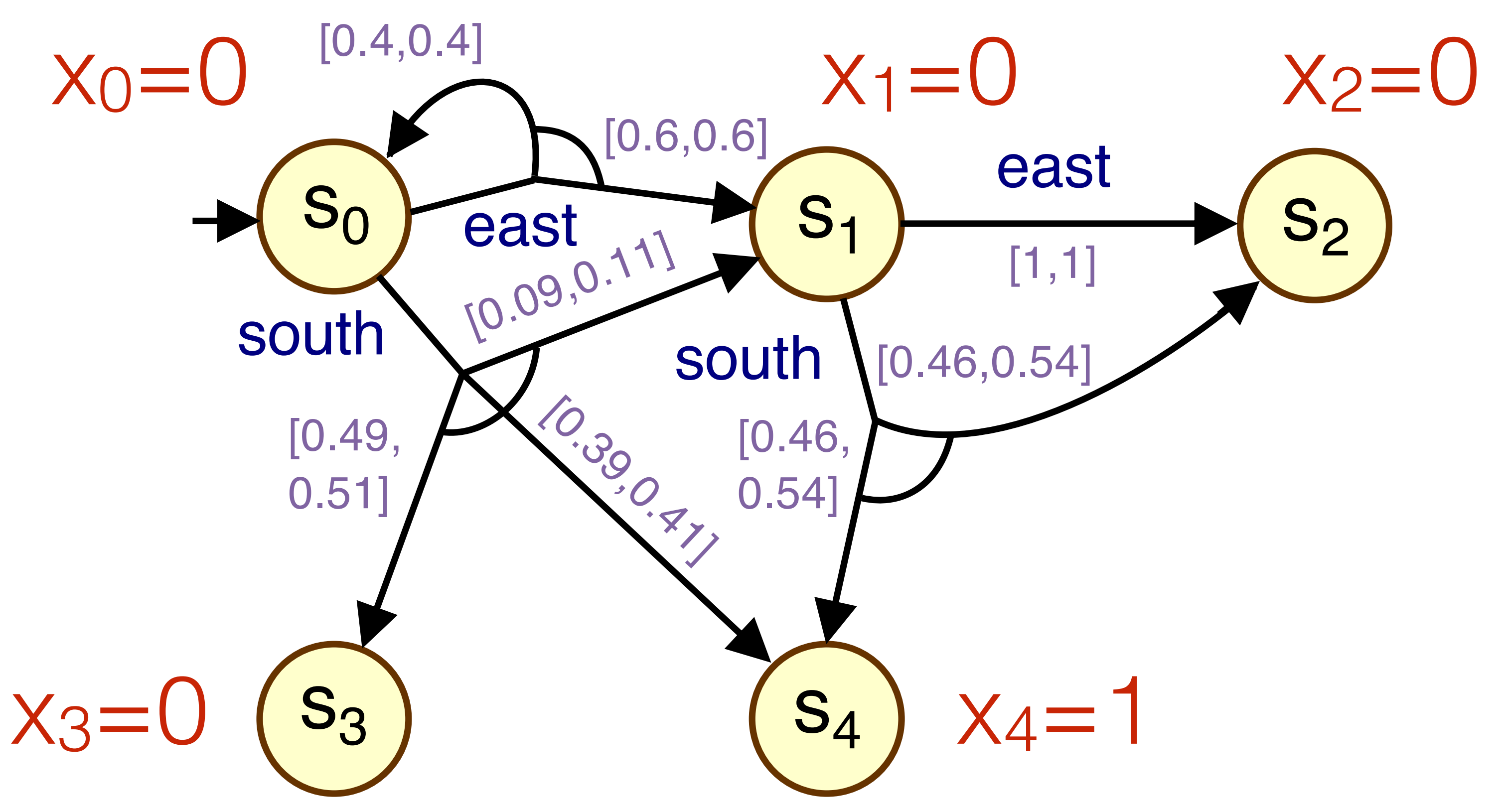
Running example: IMDPs and robust VI

- Example: MaxProb($goal_1$)



Running example: IMDPs and robust VI

- Example: MaxProb(*goal*$_1$)



- Fix $x_4=1$ and $x_2=x_3=0$, just solve for x_0, x_1
- Iteration k=0: $x_0=x_1=0$
- Iteration k=1:

$$x_0 := \max(\min(0\cdot 0.4 + 0\cdot 0.6), \min(0\cdot p_1 + 0\cdot p_3 + 1\cdot p_4))$$
$$= \max(0, 0.39)$$
$$= 0.39$$

$p_4 = 0.39, \ldots$

subject to:
$0.09 \leq p_1 \leq 0.11$
$0.49 \leq p_3 \leq 0.51$
$0.39 \leq p_4 \leq 0.41$
$p_1+p_3+p_4=1$

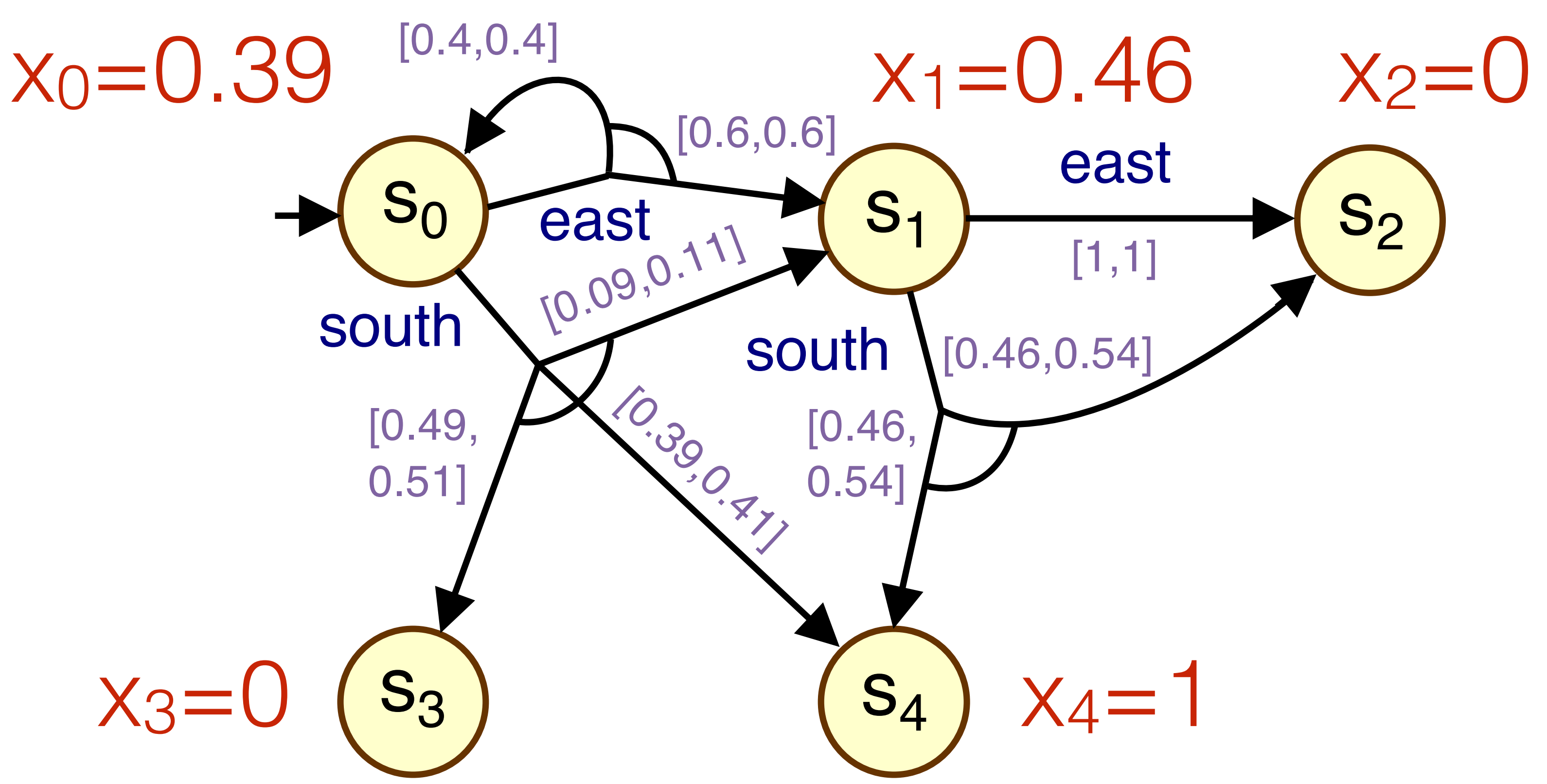
$$x_1 := \max(\min(0\cdot 1), \min(0\cdot p_2 + 1\cdot p_4))$$
$$= \max(0, 0.46)$$
$$= 0.46$$

$p_4 = 0.46, \ldots$

subject to:
$0.46 \leq p_2 \leq 0.54$
$0.46 \leq p_4 \leq 0.54$
$p_2+p_4=1$

Running example: IMDPs and robust VI

- Example: MaxProb(*goal*$_1$)



- Iteration k=2:

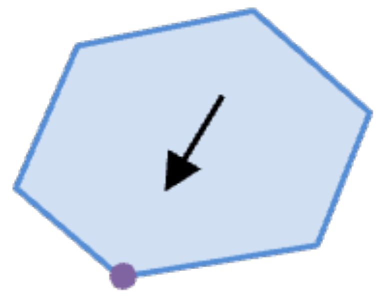
$x_0 := \max(\min(0.39 \cdot 0.4 + 0.46 \cdot 0.6),$
$\min(0.46 \cdot p_1 + 0 \cdot p_3 + 1 \cdot p_4))$

$= \max(0.432, 0.436)$
$= 0.436$

subject to:
$0.09 \leq p_1 \leq 0.11$
$0.49 \leq p_3 \leq 0.51$
$0.39 \leq p_4 \leq 0.41$
$p_1 + p_3 + p_4 = 1$

$x_3=0$
$x_1=0.46$
$x_4=1$

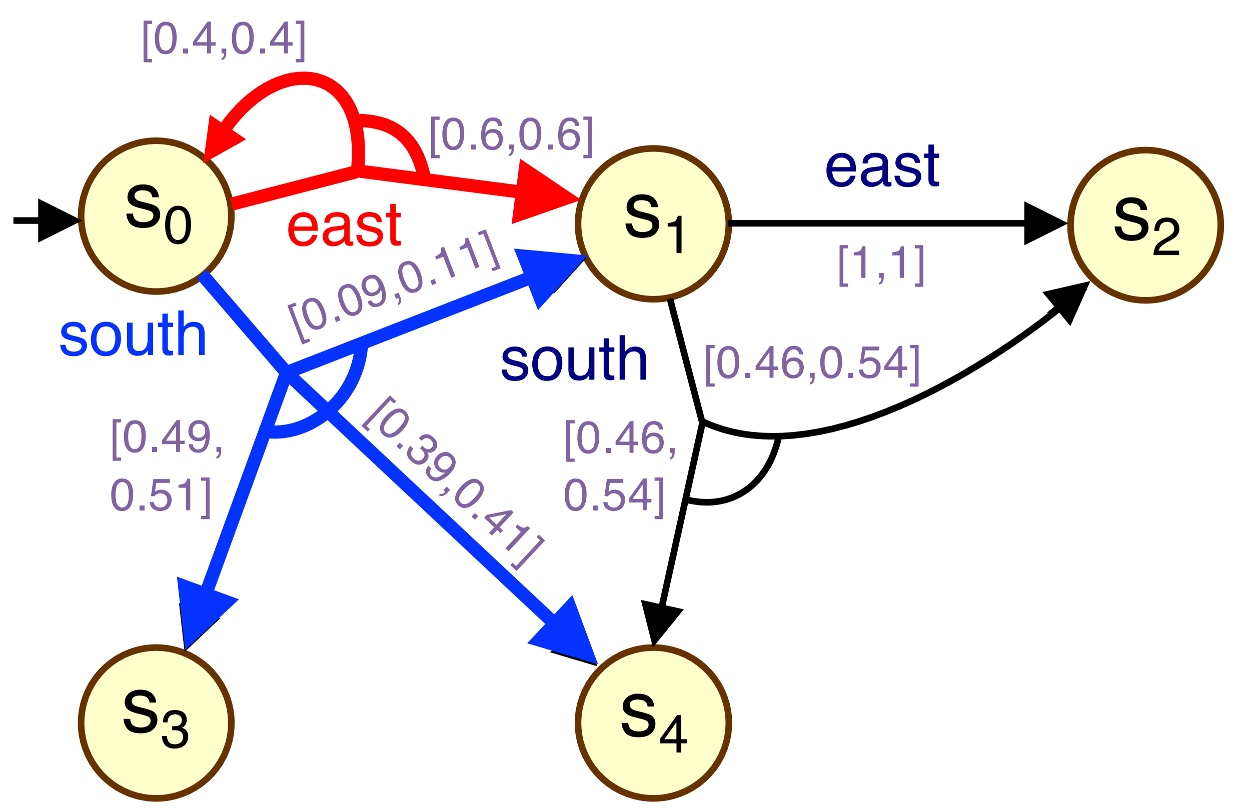
$p_3 = 0.51$
$p_1 = \min(0.11, 1-(0.51+0.39)) = 0.1$
$p_4 = 1-(0.51+0.1) = 0.39$

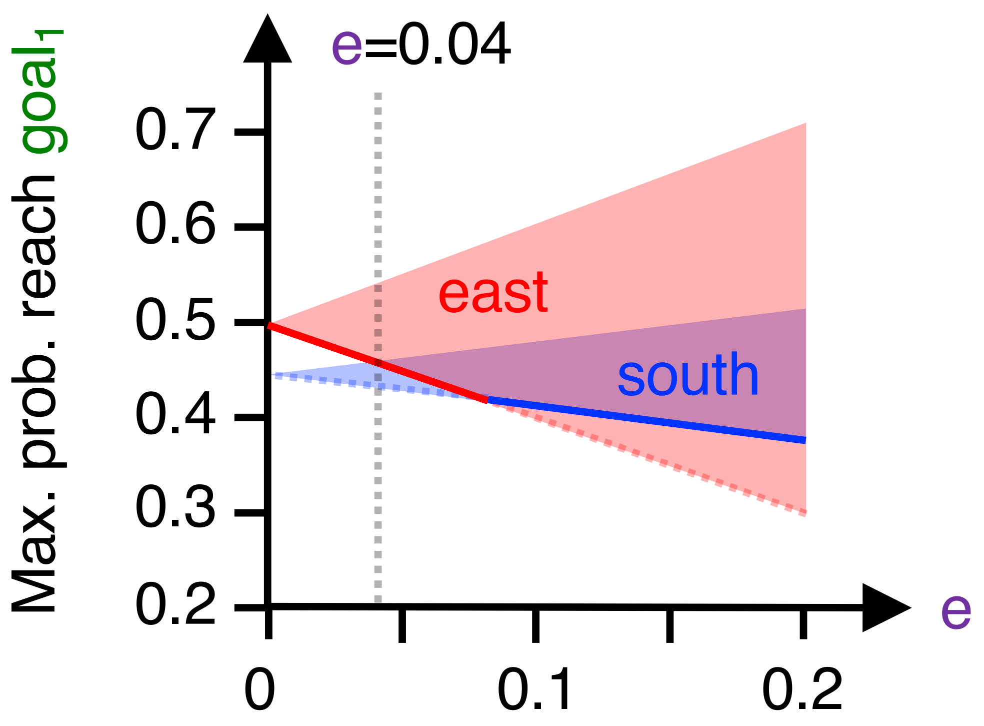


$x_1 := 0.46$ (as before)

Running example: IMDPs and robust VI

- Example: MaxProb(*goal*$_1$)





k	x_0	x_1
0	0	0
1	0.39	0.46
2	0.436	0.46
3	0.4504	0.46
4	0.45616	0.46
5	0.458464	0.46
6	0.4593856	0.46
7	0.45975424	0.46
8	0.459901696	0.46
9	0.4599606784	0.46
10	0.45998427136	0.46

- Iteration k=2:

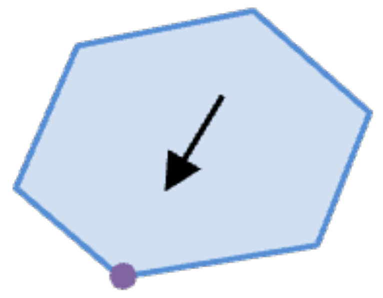
$$x_0 := \max(\min(0.39\cdot 0.4 + 0.46\cdot 0.6), \min(0.46\cdot p_1 + 0\cdot p_3 + 1\cdot p_4))$$

subject to:
$0.09 \le p_1 \le 0.11$
$0.49 \le p_3 \le 0.51$
$0.39 \le p_4 \le 0.41$
$p_1 + p_3 + p_4 = 1$

$$= \max(0.432, 0.436)$$
$$= 0.436$$

$x_3=0$
$x_1=0.46$
$x_4=1$

$p_3 = 0.51$
$p_1 = \min(0.11,\ 1-(0.51+0.39)) = 0.1$
$p_4 = 1-(0.51+0.1) = 0.39$



$x_1 := 0.46$ (as before)

- Finally: $x_0=0.46$, $x_1=0.46$

Interval MDPs - so far…

- Robust control is computationally efficient (robust value iteration)
  - (s,a)-rectangular and inner problem is easy to solve
  - another possibility not discussed here: convex optimisation [Puggelli et al.'13]
- For MaxProb (and SSP), optimal policies are memoryless (and deterministic)
  - so computed policies are optimal worst case with respect to static uncertainty

What about objectives that need memory? (e.g. finite horizon, or temporal logic)

- Intervals are a simple, natural way to model transition probability uncertainty

How do we generate the intervals?

Are there better models of uncertainty sets?

Policies with memory

- Quantifying over memoryless environment policies
  - gives us worst-case behaviour over static uncertainty

$$V^{\Pi,\mathcal{T}_m}(s) = \max_{\pi\in\Pi} \min_{\tau_m\in\mathcal{T}_m} V^{\pi,\tau_m}(s) = \max_{\pi\in\Pi} \min_{P\in\mathcal{P}} V^{\pi,P}(s)$$

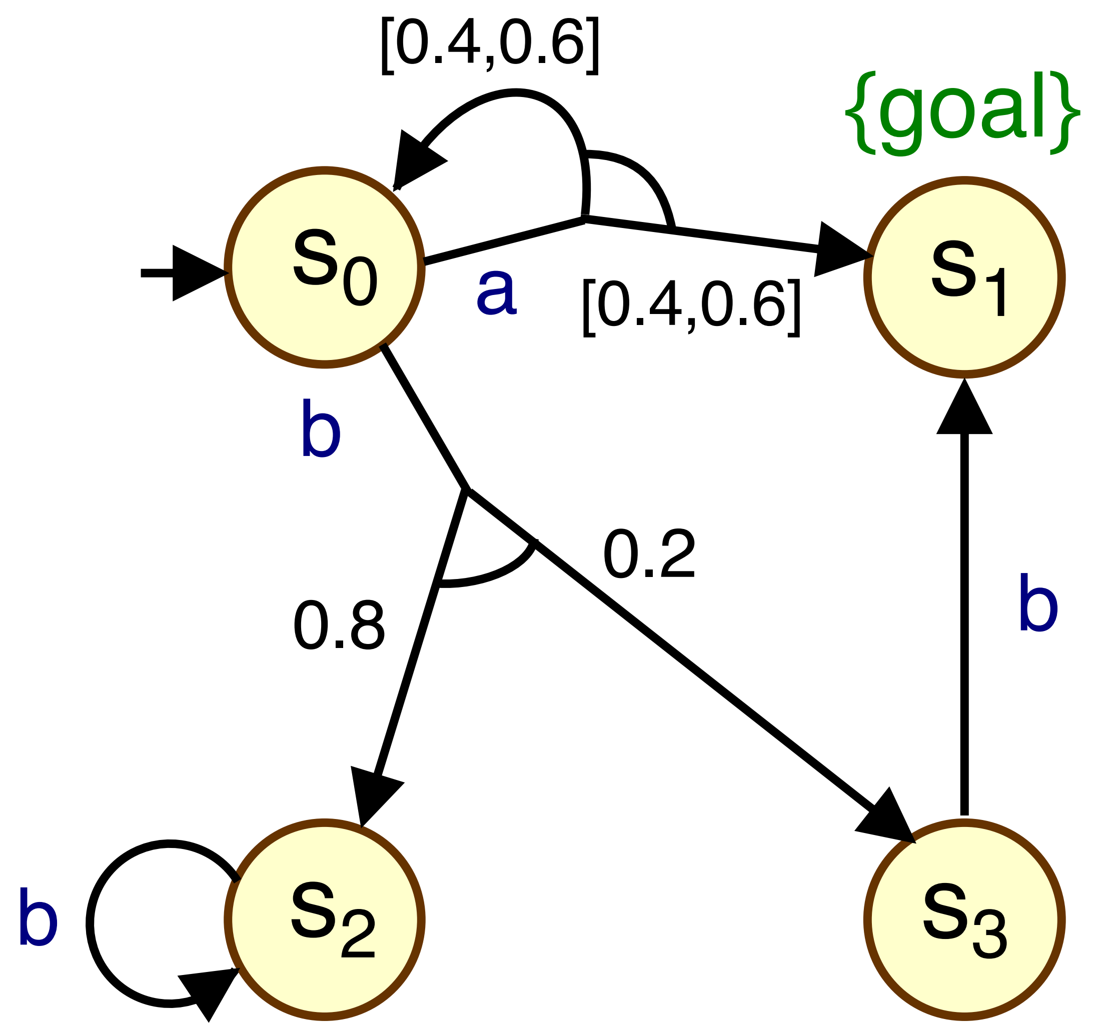
- But for objectives that require non-memoryless control policies
  - computation methods typically also assume non-memoryless environment policies

$$V^{\Pi,\mathcal{T}}(s) = \max_{\pi\in\Pi} \min_{\tau_m\in\mathcal{T}} V^{\pi,\tau_m}(s)$$

  - i.e., worst-case behaviour over dynamic uncertainty
  - which is often (but not always) unrealistic (depends on time-scales)
- This however gives a conservative bound over static uncertainty

$$V^{\Pi,\mathcal{T}}(s) \le \max_{\pi\in\Pi} \min_{P\in\mathcal{P}} V^{\pi,P}(s)$$

Memory (time dependencies)



- Objective: MaxProb$^{=2}$(goal), i.e., get to goal in <u>exactly</u> 2 steps
  - so we need time-dependent strategies for the controller
  - computable via k steps of value iteration

- Worst-case probabilities (time-dependent environment strategies)
  - "b,b" 0.2 (optimal)
  - "a,b": 0
  - "a,a": $\min\{p_1(1-p_2) \ : \ p_1, p_2 \in [0.4,0.6]\} = 0.4 \cdot (1-0.6)$ = 0.16 (too conservative)

from value iteration; dynamic uncertainty; maybe unrealistic

- Worst-case probabilities (memoryless environment strategies)
  - "b,b": 0.2
  - "a,b": 0
  - "a,a": $\min\{p(1-p) \ : \ p \in [0.4,0.6]\} = 0.4 \cdot (1-0.4)$ = 0.24 (better bound) (now optimal)

static uncertainty; may be more realistic; hard to compute

Memory (temporal logic objectives)

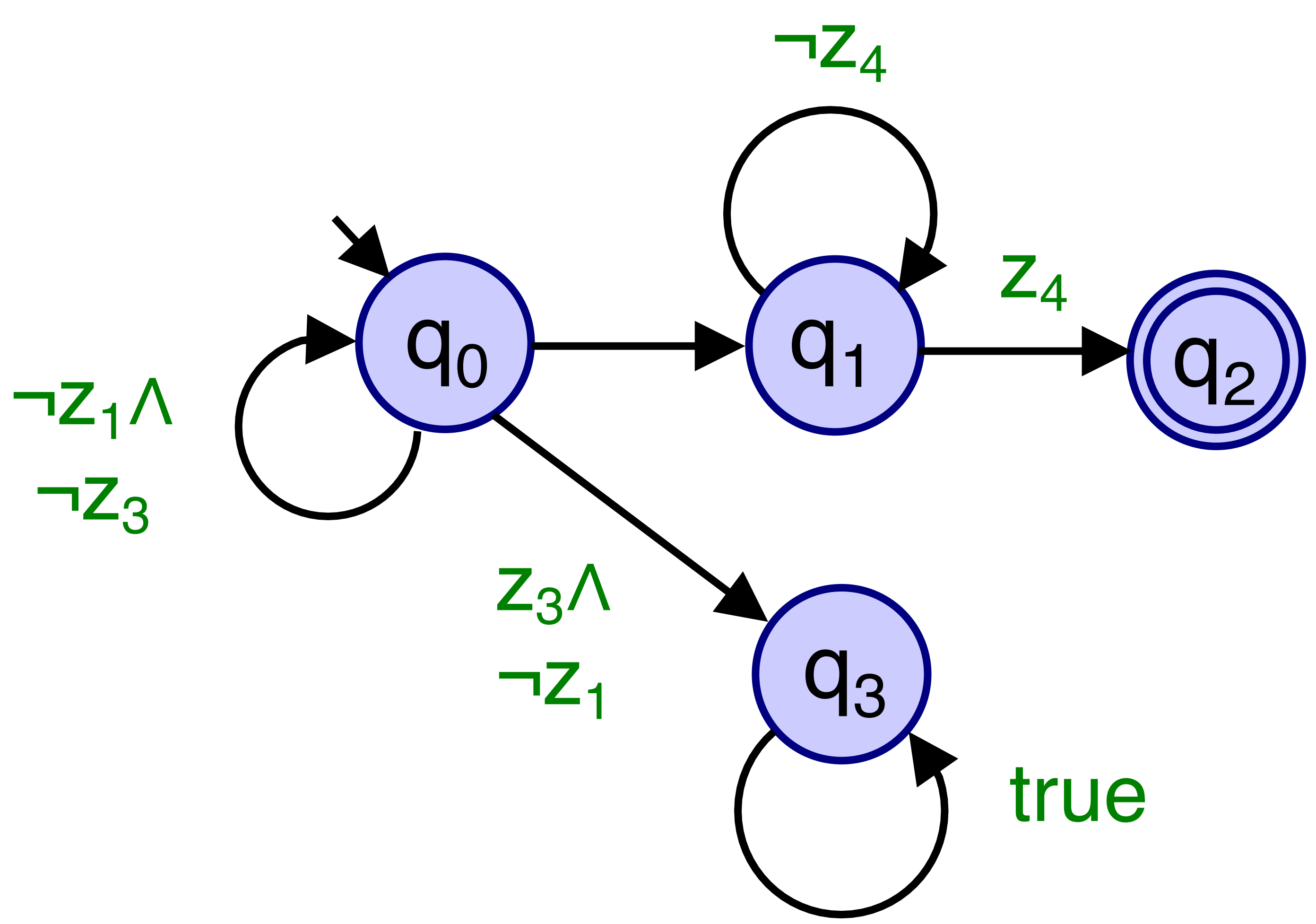
- Temporal logic (in particular LTL) allows more complex objectives, e.g.:
  - $P_{max=?}$ [(G¬hazard) ∧ (GF $goal_1$)] - “maximise probability of avoiding hazard and also visiting goal 1 infinitely often”
  - $P_{max=?}$ [¬$zone_3$ U ($zone_1$ ∧ (F $zone_4$))] - “maximise probability of patrolling zone 1 (whilst avoiding zone 3) then zone 4”
- For MDPs, we generate optimal policies by:
  - converting the LTL formula to a deterministic automaton
  - building a product of the MDP and the automaton
  - optimising a simpler objective (e.g. MaxProb) on the product MDP
- The techniques extend to uMDPs/IMDPs [Wolff et al.’12]
  - but (like for MDPs), optimal policies need memory

Automata for LTL objectives

- For co-safe LTL (satisfaction occurs in finite time), we use finite automata

$\neg zone_3 \ \mathrm{U} \ (zone_1 \wedge (\mathrm{F} \ zone_4))$

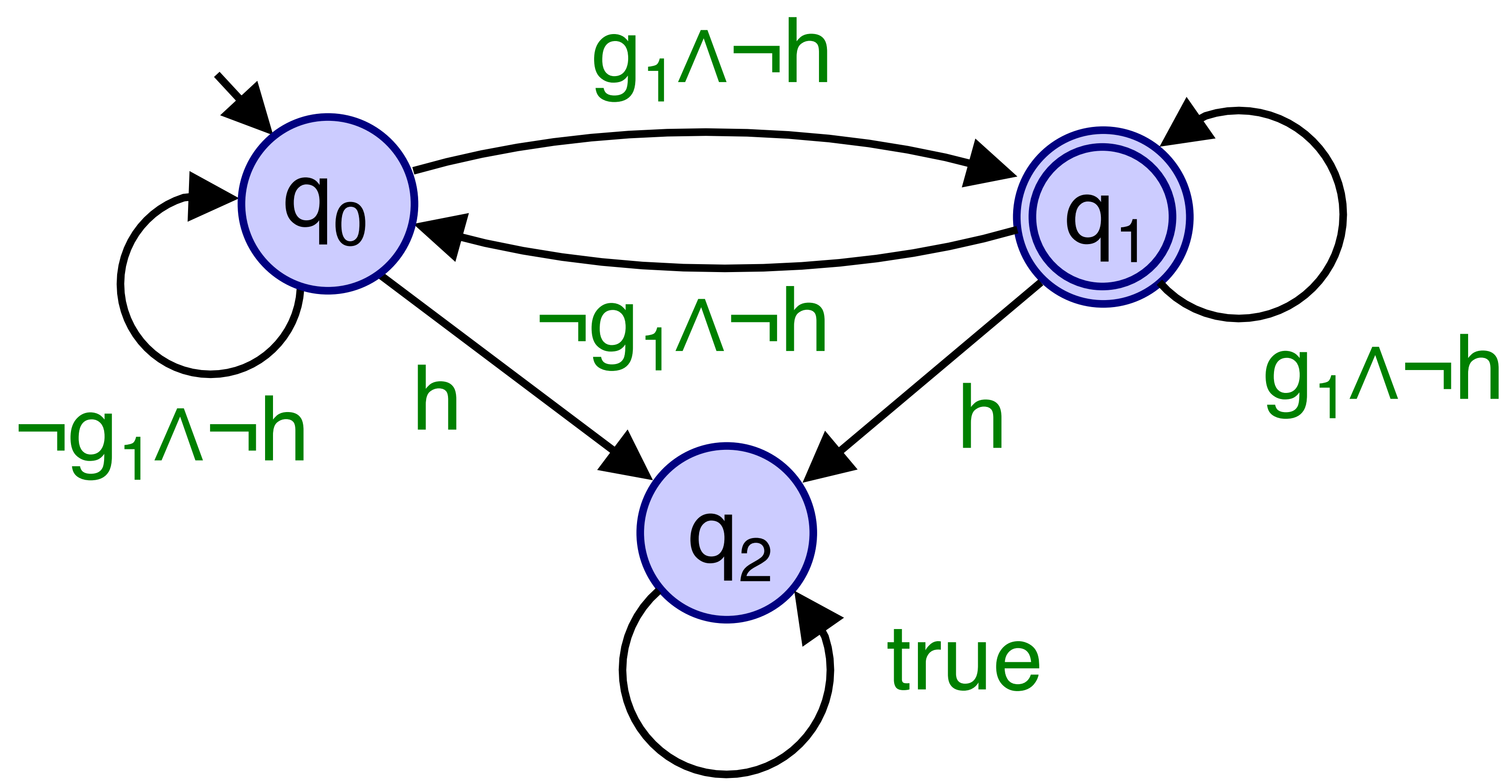
(avoiding hazard and also visiting goal 1 infinitely often)



- For general LTL, we use e.g. Rabin automata

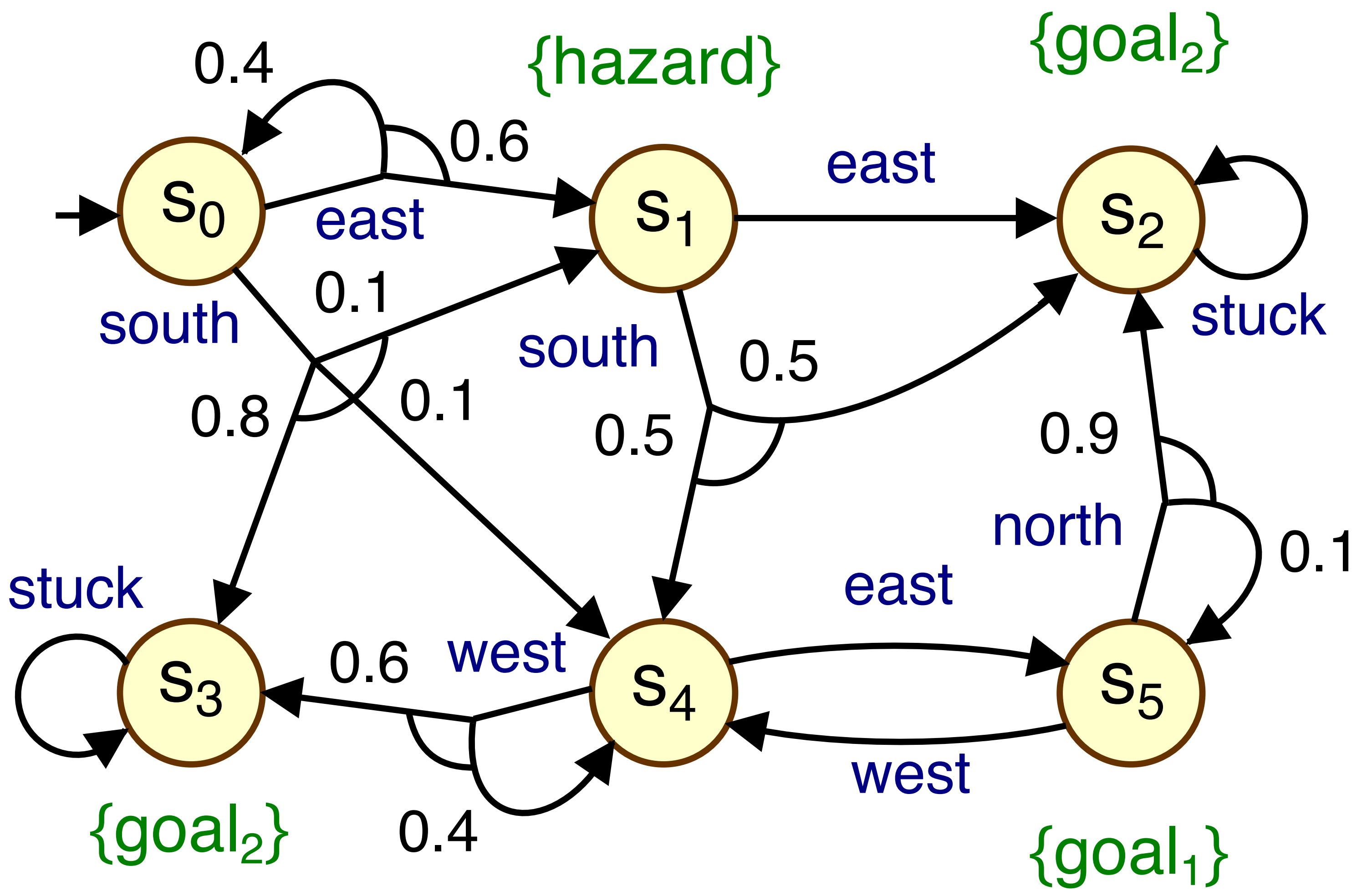
$(\mathrm{G} \neg hazard) \wedge (\mathrm{GF} \ goal_1)$

(visit zone 1 (whilst avoiding zone 3) then zone 4)

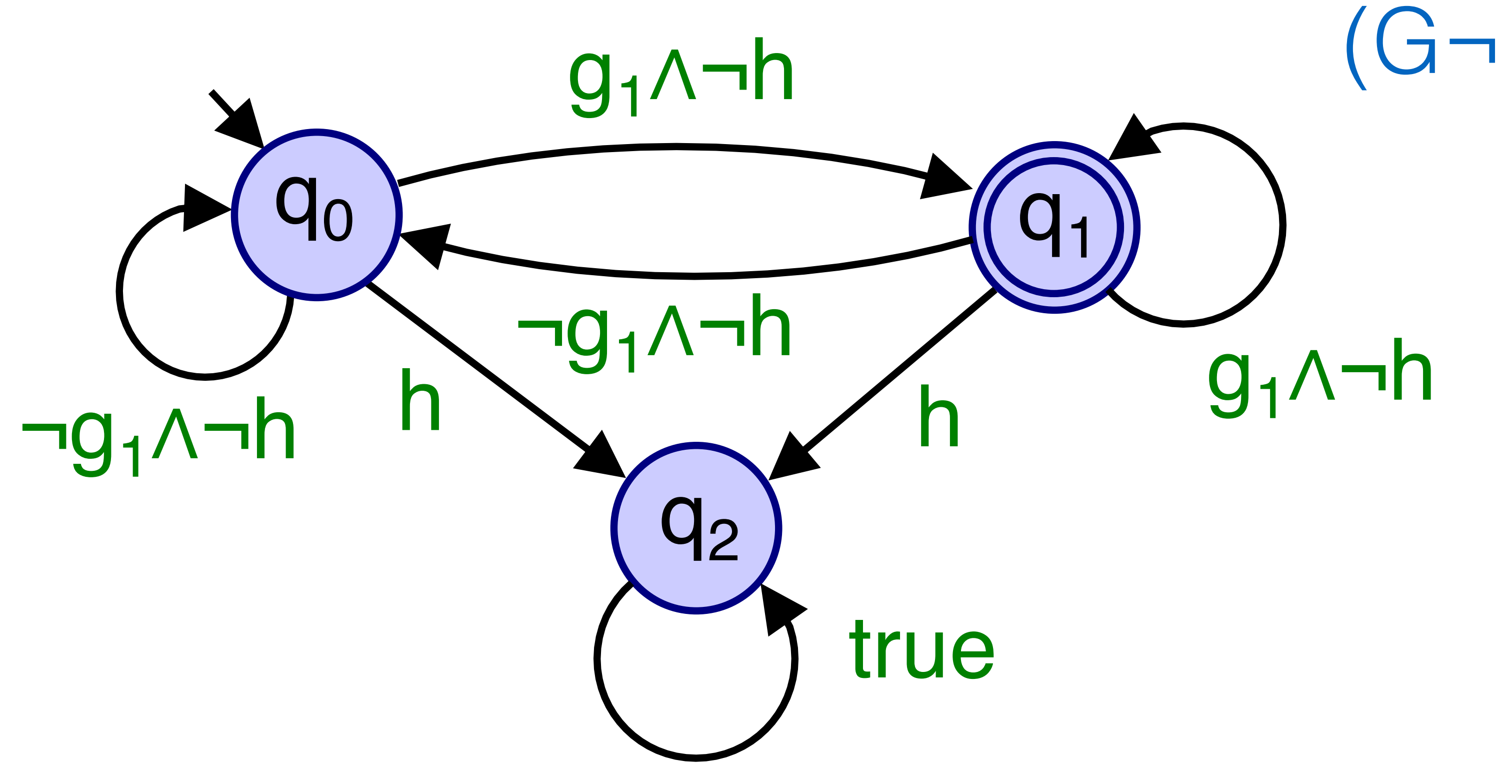


Optimising for LTL on a product MDP

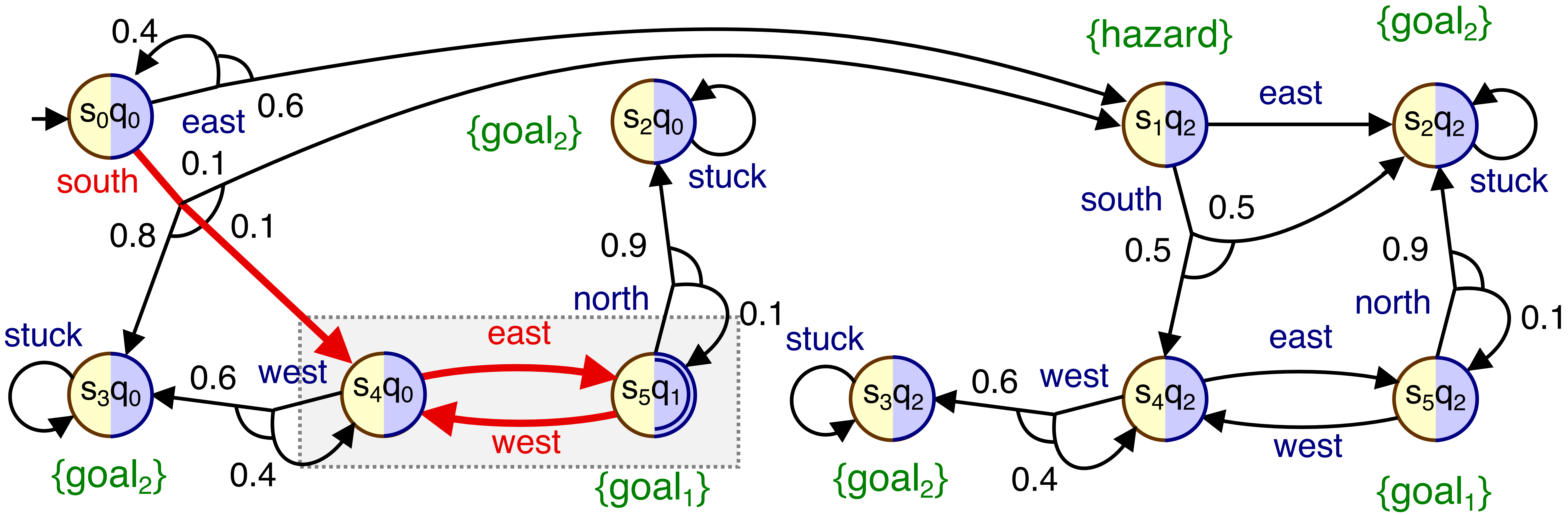
MDP M



Automaton $\mathscr{A}$ for
(G¬hazard) ∧ (GF goal$_1$)



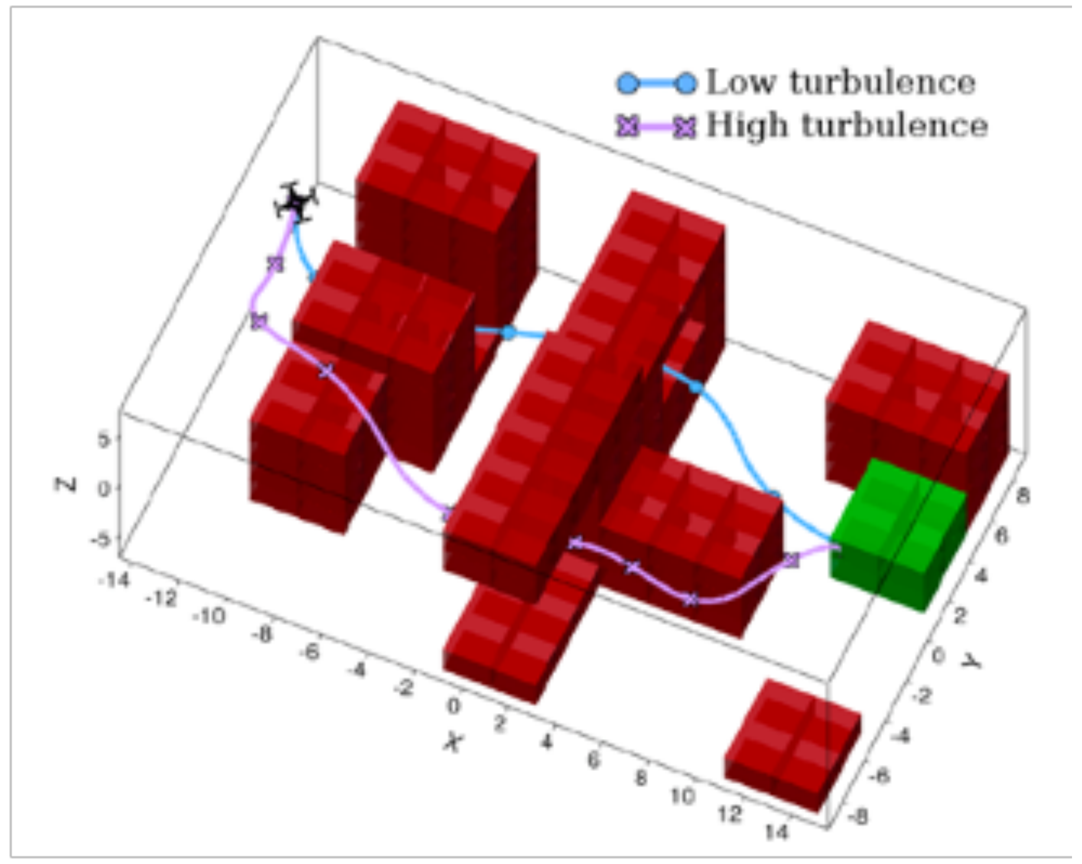
Product MDP $M \otimes \mathscr{A}$

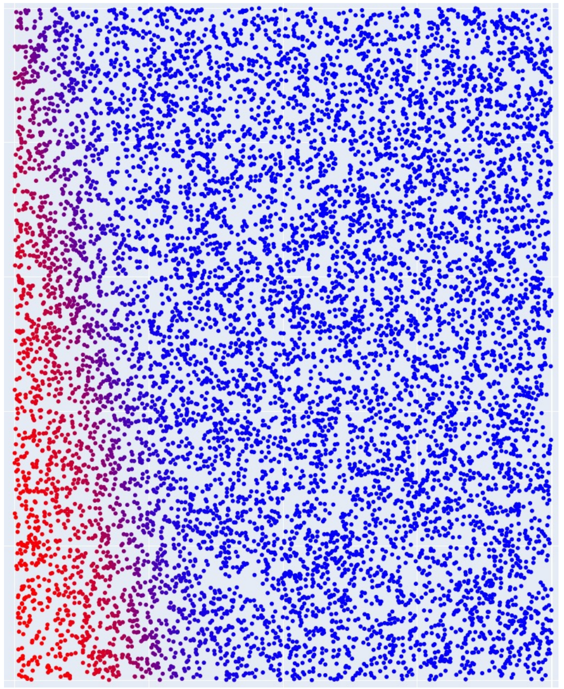


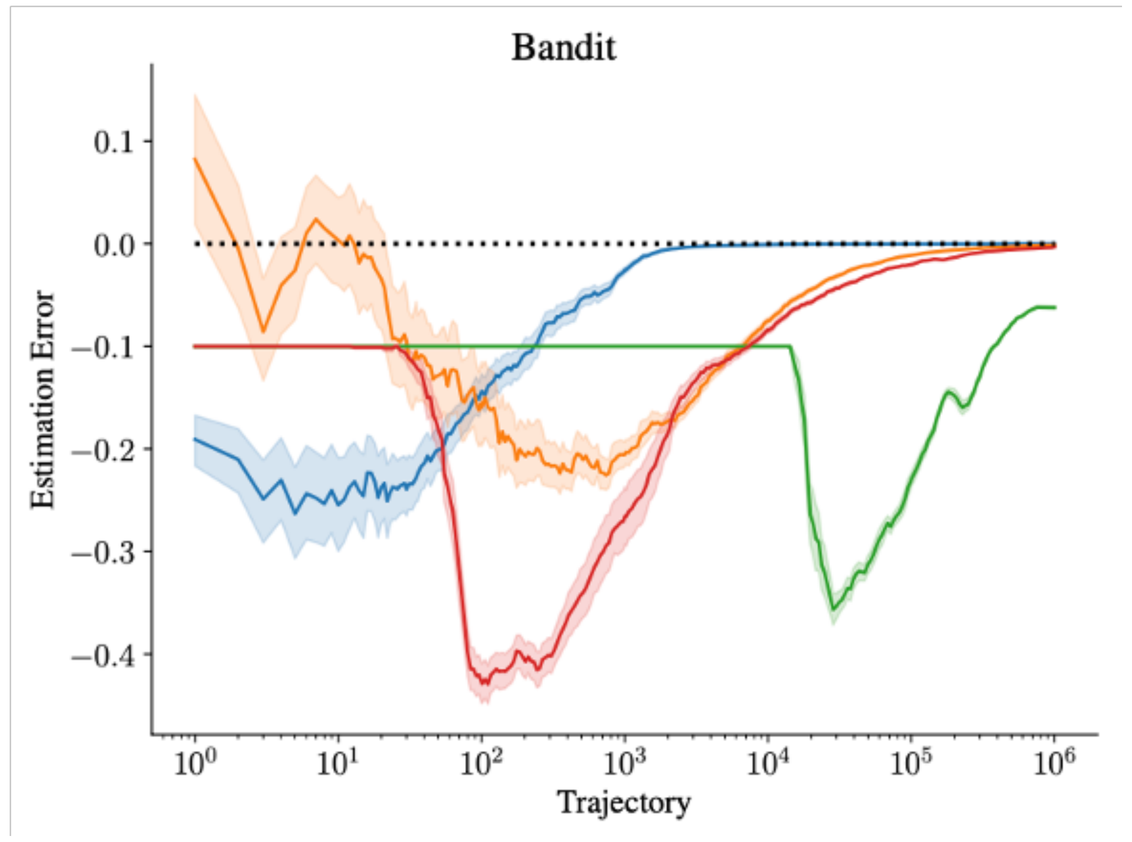
Optimal memoryless policy of $M \otimes \mathscr{A}$ corresponds to finite-memory optimal policy of MDP M

Generating IMDP intervals

- Some examples of IMDP generation







- Unmanned aerial vehicle
  - robust control in turbulence
  - continuous-space dynamical model with unknown noise
  - discrete abstraction + finite “scenarios” of sampled noise yields IMDP abstraction

[Badings et al.’23]

- Deep reinforcement learning
  - worst-case analysis of abstractions of probabilistic policies for neural networks
  - intervals between IMDP abstract states constructed by sampling the policy

[Bacci&Parker’20]

- Robust anytime MDP learning
  - sampled MDP trajectories
  - IMDPs constructed and solved periodically to yield robust predictions on current model
  - PAC or Bayesian interval learning

[Suilen et al.’22]

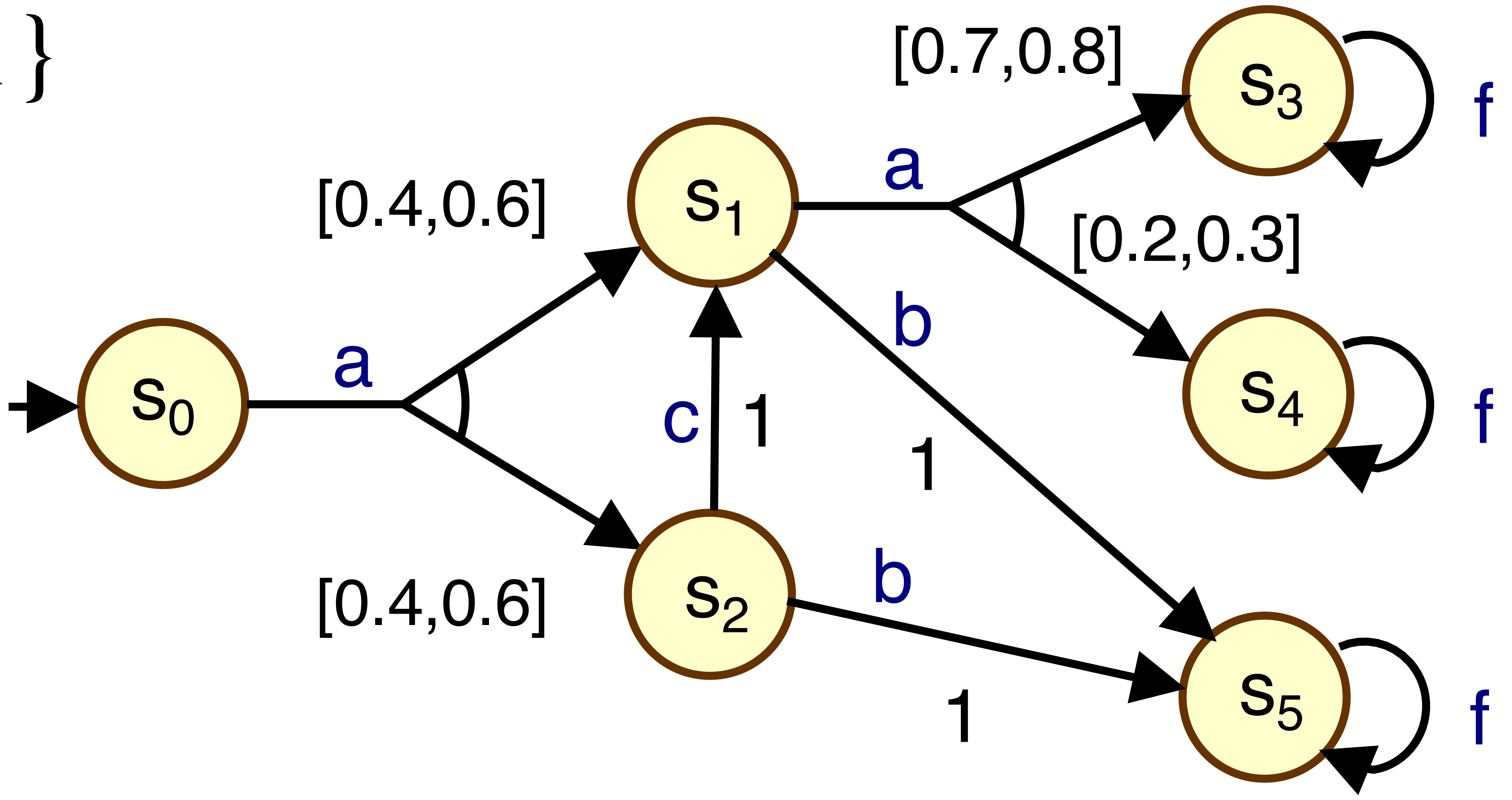
Learning IMDP intervals

- One approach: sampling from the (fixed, but unknown) “true” MDP
  - generate sample paths and keep separate counts of transition frequencies
- Gives confidence intervals around point estimates for transition probabilities $P_s^a(s_i)$
  - using probably approximately correct (PAC) guarantees
  - we fix an error rate γ and compute an error δ
  - standard method of maximum a-posteriori probability (MAP) estimation to infer point estimates of probabilities
- For each state s, we have sample counts $N = \#(s, a)$ and $k_i = \#(s, a, s_i)$
  - point estimate of the transition probability $P_s^a(s_i)$ is: $\tilde{P}_s^a(s_i) \approx k_i / N$
  - confidence interval for the transition probability: $\tilde{P}_s^a(s_i) \pm \delta$ where $\delta = \sqrt{\log(2/\gamma)/2N}$
  - then we have: $Pr(P_s^a(s_i) \in \tilde{P}_s^a(s_i) \pm \delta) \geq 1 - \gamma$ (via Hoeffding’s inequality)

Learning IMDP intervals

- If desired, we can lift the PAC guarantee from individual transitions to the uMDP

- Distribute the chosen error rate γ across all transitions:
  - $\gamma_P = \gamma / (\Sigma(s,a) \in S \times A \,|\, Succ_{>1}(s,a) \,|\,)$
  - where $Succ_{>1}(s,a) = \{s \in S : 0 < P_s^a(s') < 1\}$ is the set of successor states of each (s,a) with more than one successor



- To construct the IMDP, we use:
  - $\underline{P}_s^a(s_i) = \max(\varepsilon, \tilde{P}_s^a(s_i) - \delta_P)$
  - $\overline{P}_s^a(s_i) = \min(\tilde{P}_s^a(s_i) + \delta_P, 1)$

- Then we have: $Pr(P \in \mathscr{P}) \geq 1 - \gamma$

[Suilen et al.'22]

Likelihood uncertainty sets

[Nilim&Ghaoui'05]

- Likelihood models suit experimentally determined transition probabilities
  - and are less conservative than interval representations

- Uncertainty sets are :
  - are derived from empirical frequencies $F_s^a(s')$ of a transition to s' after action a in state s
  - are described by likelihood regions: $\mathscr{P}_s^a = \{P_s^a \in Dist(S) \mid \sum_{s'} F_s^a(s')\log(P_s^a(s')) \geq \beta_s^a)\}$
  - where β_s^a is the uncertainty level (can be estimated for a desired confidence level)
  - $\beta_s^a < \beta_{s,\max}^a$ where $\beta_{s,\max}^a = \sum_{s'} F_s^a(s')\log(F_s^a(s'))$ is the optimal log-likelihood

- Inner optimisation problems
  - can be solved (approximately) using a bisection algorithm
  - to within an accuracy δ in time $O(\log(x_{\max}/\delta))$ where $x_{\max}$ is the maximum value in vector x

$$\inf_{P_s^a \in \mathscr{P}_s^a} \sum_{s' \in S} P_s^a(s') \cdot x_{s'}$$

Uncertainty set models - Summary

- Intervals & likelihood models
  - both quite computationally tractable and statistically meaningful
  - interval models are more conservative (sometimes projected to as an estimate)
- Finite scenarios (“sampled”): $\mathscr{P}_s^a = \{P_{s,1}^a, \ldots, P_{s,k}^a\}$
  - inner optimisation is simple (min over finite set)
  - but worst-case choice can be very conservative

$$\inf_{P_s^a \in \mathscr{P}_s^a} \sum_{s' \in S} P_s^a(s') \cdot x_{s'}$$

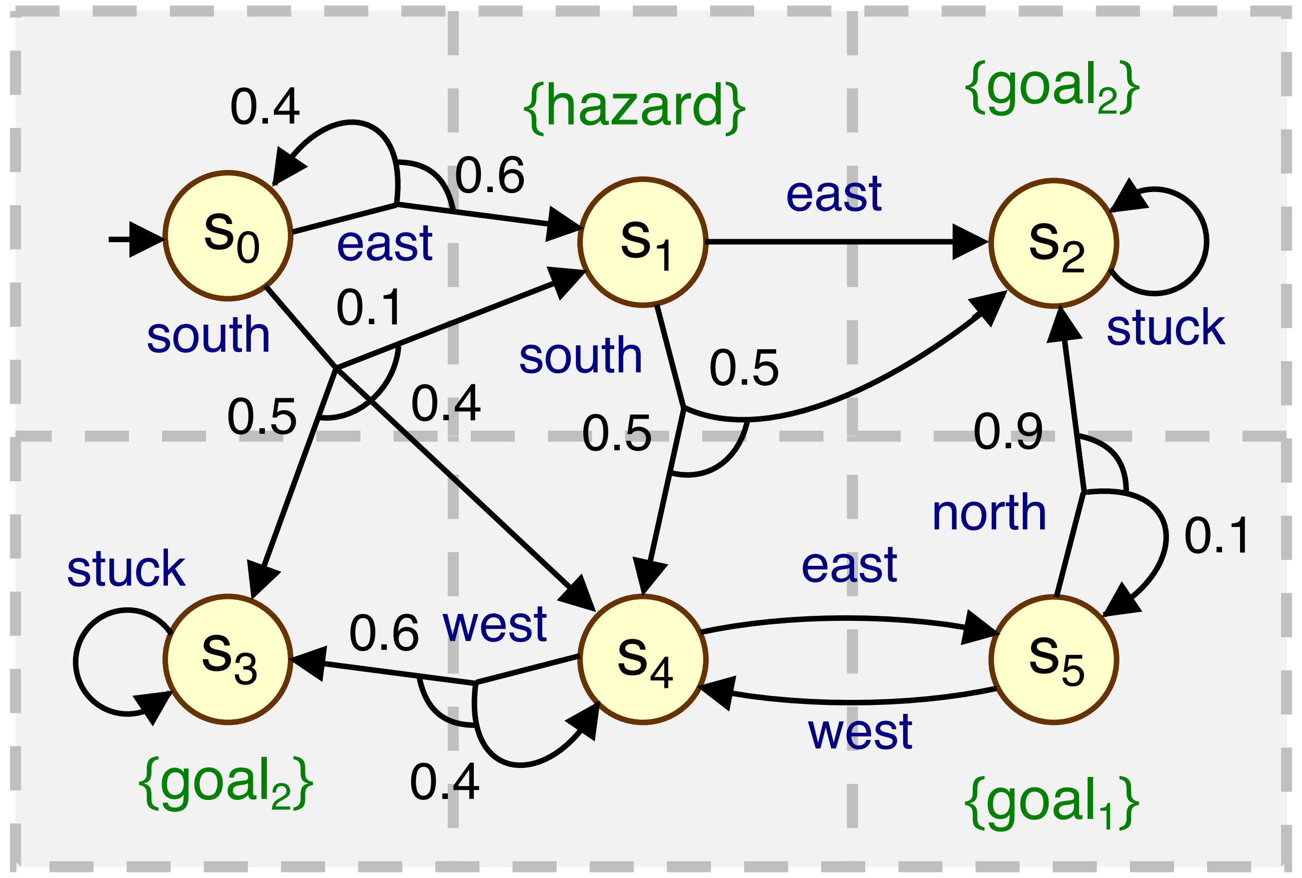
- Many other possibilities, e.g.:
  - maximum a posteriori models, entropy models, ellipsoidal models, …
  - most have similar (approximate) optimisation approaches to likelihood models
  - see: [Nilim&Ghaoui’05] for details

Tool support: PRISM

- PRISM: probabilistic model checking tool
  - formal modelling and analysis (using temporal logic properties) of:
    - Markov chains, Markov decision processes,
    - interval Markov chains, interval Markov decision processes,
    - stochastic games (via PRISM-games), and much more…

PRISM

- See: www.prismmodelchecker.org
  - download, documentation, tutorials, papers, case studies, …



- Supporting files for ESSAI examples here:

  www.prismmodelchecker.org/courses/essai23/

Advertisement

- ERC-funded project FUN2MODEL, based at Oxford
  - lead by Marta Kwiatkowska
  - model-based reasoning for learning and uncertainty
- Postdoc position available now
  - http://www.fun2model.org/
  - http://www.prismmodelchecker.org/news.php

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Summary (part 3)

- Uncertain MDPs
  - environment policies - static vs dynamic uncertainty
  - robust value iteration (robust dynamic programming)
  - implementation with interval MDPs (IMDPs)
  - non-memoryless policies (static uncertainty)
  - generating / learning intervals
  - uncertainty set representations
  - tool support: PRISM

- Up next: Sampling-based uncertain MDPs
  - removing the transition independence assumption (rectangularity)

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